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Changing Beliefs and Systematic Rational Forecast Errors with Evidence from Foreign Exchange

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Recent evidence concerning dollar forecasts during the early 1980s have led to assertions that the market was irrational. This paper investigates an alternative interpretation. Following the tightening of the U.S. money market, agents did not immediately believe that the change would persist, but instead learned the shift rationally. Empirical simulations indicate that the model appears consistent with about half of the dollar's underprediction implied by the forward market during the period.

According to the "Rational Expectations" paradigm, the market uses efficiently all available information in forming forecasts of future variables. Assuming also that the market knows the underlying distribution of economic disturbances, this paradigm implies that forecast errors are uncorrelated with the information set used to form the forecasts. Under this additional assumption, the paradigm of rational expectations, used extensively throughout macroeconomics, has come to be associated with the presumption that forecast errors have mean zero.

Recent empirical evidence from the behavior of one macroeconomic variable, the exchange rate, has suggested a potential contradiction to this implication of rational expectations. For example, on the basis of survey data Jeffrey Frankel and Ken Froot (1987) find that market participants systematically underpredicted the strength of the dollar during the early 1980s. Furthermore, the prediction of the forward dollar exchange rate implied a weaker dollar than was realized on-average from the period from 1980 through 1985.[1] Therefore, some interpret the overall evidence of systematic dollar forecast errors as evidence of market irrationality.

By contrast, this paper investigates a different source of systematic forecast errors, where agents in fact use all available information efficiently and in this sense are rational. In general, the paper analyzes the forecast error effects due to a change in the process of fundamentals that the market learns only over time using Bayesian updating.[2] In particular, this framework is used to empirically investigate the implied impact upon dollar forecast errors due to learning about the increase in U.S. money demand in

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[1] Richard Levich (1985) shows that these "excess returns" on dollar assets were statistically significant and ranged from a monthly rate of 0.6 percent for the Japanese yen to 1.4 percent for the Swiss franc during this period. Although a risk premium against the dollar could theoretically explain this behavior, the period of largest excess returns began in 1981, at a time when the market analysts referred to the dollar buying by foreigners as "a flight to quality" and as a "safe haven." See Robert Cumby (1988).

[2] This behavior is similar to the systematic surprises to the Phillips curve and, hence, employment as an economy converges to equilibrium, as described in John Taylor (1975). In a related issue, Robert Flood and Peter Garber (1980) and Marianne Baxter (1985) study agents' beliefs about the credibility of government reforms using Bayesian methods.

the early 1980s. Using conservative values for the range of plausible parameter values, this learning model implies systematic under-prediction of the dollar's strength by about one-half the levels suggested by the forward exchange rate.

This paper focuses upon the shift in U.S. money demand for three reasons. First, at the time of this shift the Federal Reserve used a non-borrowed reserves target for monetary policy, a procedure that does not allow changes in money demand to be fully accommodated. Second, since the increase in money demand appeared to surprise the Federal Reserve as well as the private sector, it seems plausible to suppose that the increase in money demand was not fully anticipated. Third, unlike more model-specific exchange rate determinants, money demand affects the exchange rate in the same manner for a wide class of models.

In the paper, the exchange rate effects from learning about higher money demand are calculated based upon two polar assumptions about the market's knowledge of the new money demand equation. First, in the event that the money demand equation has changed, the market knows the parameters of the new equation. Second, the market learns the parameters of the new equation only over time.

The paper is organized as follows. Section I describes the behavior of systematic forecast errors for a general forward-looking asset price when the market learns about a change in a fundamentals process. Section II relates this analysis to the dollar exchange rate due to an increase in the process of U.S. money demand when the market knows the parameters of the new process. Section III investigates the effects upon the forecast errors when the market does not know the new parameters, but only learns them over time. Concluding remarks follow in Section IV.

I. Systematic Forecast Errors and Evolving Beliefs

The following simple example demonstrates how forecast errors may be systematically incorrect while the market rationally learns the true process that generates fundamentals. This example represents in general the behavior of prices that have forward-looking solutions, such as stock prices (Robert Shiller, 1981) or hyperinflation (Tom Sargent and Neil Wallace, 1973). Despite its general representation, this variable will be called the "exchange rate" since the analysis will be applied to the U.S. dollar exchange rate in the following section.

To motivate the behavior of forecast errors, suppose the exchange rate is determined by a set of fundamental variables that influence the demand for and supply of currency at each point in time and by the expected future exchange rate.[3] In particular, s_t, the logarithm of the exchange rate, is given by the following simple equation:

$$(1)\qquad s_t = n_t - z_t + \alpha E_t(s_{t+1} - s_t),$$

where $E_t(\cdot)$ is the conditional expectations operator and where z_t and n_t are "fundamentals" variables that determine the exchange rate with coefficients that have been arbitrarily set equal to 1 and -1, respectively. While the distribution of n_t is assumed stationary and ergodic throughout, the process for z_t may switch from one process to another (as discussed below). To focus upon the market's beliefs about this switch, z_t and n_t are assumed uncorrelated. Solving equation (1) forward gives the solution of the exchange rate in terms of future expected "fundamentals":

$$\begin{aligned}(2)\quad s_t &= (1/(1+\alpha)) \\ &\qquad \times \sum_{j=o}^{\infty} (\alpha/(1+\alpha))^j E_t(n_{t+j} - z_{t+j}) \\ &\equiv {}_tN_t - (1/(1+\alpha)) \\ &\qquad \times \sum_{j=o}^{\infty} (\alpha/(1+\alpha))^j E_t(z_{t+j}),\end{aligned}$$

where

$$_rN_k \equiv (1/(1+\alpha))\Sigma_{j=o}^{\infty}(\alpha/(1+\alpha))^j \times E_r(n_{k+j}).$$

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[3] See Jacob Frenkel and Michael Mussa (1980), for example.

Since the n_t are stationary, they have a time-series representation with white-noise i.i.d. innovations.

Before describing the effects of revising beliefs about the distribution of z_t, consider first the exchange rate forecast errors under the standard assumption that the market knows with certainty the process followed by the fundamentals. Furthermore, assume that the z_t process is stationary after first-differencing and is given by,

$$\Delta z_t = \delta_o + v_{o,t}, \tag{3}$$

where Δ is the difference operator, δ_o is a constant parameter, and $v_{o,t}$ is a white-noise, normally distributed disturbance term. Taking the expectations of future values of z_t and n_t and substituting the result into equation (2) gives the following exchange rate solution:

$$s_t = -v_{o,t} - (1+\alpha)\delta_o - z_{t-1} + {}_tN_t. \tag{4}$$

Taking the conditional expectation operator across equation (4) gives the mean zero i.i.d. forecast errors:

$$s_t - E_{t-1}s_t = -v_{o,t} + ({}_tN_t - {}_{t-1}N_t). \tag{5}$$

Now suppose that at a point in time, τ, market participants believe that the process of z_t may have changed due to an event or announcement exogenous to the process of fundamentals. Suppose further that if the process in equation (3) in fact changed at time τ, the market knows that this new process will follow,

$$\Delta z_t = \delta_n + v_{n,t}, \quad \text{for } t \geq \tau, \tag{3$'$}$$

where $\delta_n > \delta_o$ and where $v_{n,t}$ is a white-noise, normally distributed random variable. In general, the increase in δ represents a switch in the fundamentals process that strengthens the exchange rate.

Over time, the market would discover the true process, parameterized either by δ_o if there is no change, or by δ_n if it changed to the new process. To characterize this learning process, market participants are assumed to form Bayesian forecasts, assigning a probability weight to either process. The market's uncertainty about the process followed by z_t will affect the exchange rate as in equation (2) by altering the present and future expectations of z_t. That is, the expected future values of this fundamental variable are probability-weighted averages of the two processes in equations (3) and (3′), respectively. Specifically, defining $P_{j,t}$ as the probability that the process generating z_t has the parameter δ_j, for $t > \tau$,

$$\begin{aligned} (6) \quad & E_{t-1}(\Delta z_{t-1+j}) \\ &= P_{o,t-1}\delta_o + P_{n,t-1}\delta_n, \qquad j > 0 \\ &= (P_{o,t-1}\delta_o + P_{n,t-1}\delta_n) \\ &+ (P_{o,t-1}\hat{v}_{o,t-1} + P_{n,t-1}\hat{v}_{n,t-1}) \equiv \Delta z_{t-1}, \\ & \qquad j = 0, \end{aligned}$$

where $P_{o,t} + P_{n,t} = 1$ and where $\hat{v}_{j,t} = \Delta z_t - \delta_j$, the market's estimate of the current disturbance given that j is the process. Clearly, since the $v_{j,t}$ are white noise, the expected future value of the fundamental variable is a simple probability-weighted average of the two δ parameters. The market decomposes the current observation of Δz_t into two components implied by each process.

Substituting the expected future fundamentals in equation (6) into the exchange rate solution in equation (2), implies the following form:

$$\begin{aligned} (7) \quad s_t = & -[P_{o,t}\hat{v}_{o,t} + P_{n,t}\hat{v}_{n,t}] \\ & -(1+\alpha)(P_{o,t}\delta_o + P_{n,t}\delta_n) \\ & - z_{t-1} + {}_tN_t. \end{aligned}$$

Furthermore, subtracting from equation (7) the exchange rate forecast conditional upon $t-1$ information gives the market's forecast error based upon their updated beliefs about the process of z.

$$(8)\quad \left(s_t - E_{t-1}(s_t|P_{j,t-1})\right) = -\left[P_{o,t}\hat{v}_{o,t} + P_{n,t}\hat{v}_{n,t} + (1+\alpha)(\delta_n - \delta_o)(P_{n,t} - P_{n,t-1})\right] + ({}_tN_t - {}_{t-1}N_t) = -e_t - \hat{v}_{n,t} + ({}_tN_t - {}_{t-1}N_t),$$

where $e_t \equiv P_{o,t}(\hat{v}_{n,t} - \hat{v}_{o,t}) + (1+\alpha)(\delta_n - \delta_o)\Delta P_{n,t} \equiv e_{1,t} + e_{2,t}$. Since the ${}_tN_t$ terms have mean zero and are uncorrelated with z_t, any systematic behavior in the forecast errors must arise from the component that depends upon z_t. Hence, without any loss of generality, the forecast errors, ${}_tN_t - {}_{t-1}N_t$, will be set equal to zero for the remainder of the paper.

To investigate the behavior of the component due to changing beliefs about the shifting fundamental process requires further specifying how the market updates beliefs about the process. At the initial point in time τ, market participants assign a probability, $P_{n,\tau}$, to the event that δ switched from δ_o to δ_n. Thereafter, they update this probability based upon subsequent observations according to Bayes' law. Thus,

$$(9)\quad \frac{P_{n,t} = P_{n,t-k}f(\Delta z_t,\ldots,\Delta z_{t-k}|\delta_n)}{[P_{n,t-k}f(\Delta z_t,\ldots,\Delta z_{t-k}|\delta_n) + P_{o,t-k}f(\Delta z_t,\ldots,\Delta z_{t-k}|\delta_o)]^{-1}},$$

where $f(\Delta z_t|\delta_i)$ is the density function of Δz_t given δ_i and where $P_{j,t-k}$ are the prior probabilities at some lag k. Clearly, the market's beliefs about the process move over time in response to realizations of the random variable z. Asymptotically, the probability assigned to the new process, P_n, converges either to one, if in fact the process has changed, or to zero if no change has taken place.[4] That is, if the true parameter of the process generating z_t is δ_i, then plim $P_{i,t} = 1$.

Even though the forecasts minimize the market's errors conditional upon their prior beliefs, the expected value of the market's Bayesian forecast errors based upon this true distribution of z_t will not in general be zero during the period while market participants are learning. For example, suppose that in fact the process of z_t changed to the "new" process given by δ_n at time τ. Then, from equation (8), any nonzero expectation of the forecast errors based upon this true distribution depends only upon the expected value of e_t, since: $E(\hat{v}_{n,t}|\delta_n) = E(v_{n,t}) = 0$. Therefore, based upon the true process for z_t, a sample mean of the exchange rate forecast errors conditional upon the beliefs embodied in $P_{j,t}$ can be written as decomposed into $e_{1,t}$ and $e_{2,t}$.

$$(10)\quad E\left\{(1/T)\left[\sum_{t=\tau}^{T+\tau}\left(s_t - E_{t-1}(s_t|P_{j,t-1})\right)\right]|\delta_n\right\} = -E\left\{(1/T)\left[\sum_{t=\tau}^{T+\tau} e_{1,\tau}\right]|\delta_n\right\} - E\left\{(1/T)\left[\sum_{t=\tau}^{T+\tau} e_{2,\tau}\right]|\delta_n\right\} = -E\left\{(1/T)\sum_{t=\tau}^{T+\tau}(P_{o,t}(\hat{v}_{o,t} - \hat{v}_{n,t}))|\delta_n\right\} - (1+\alpha)E\left\{(1/T)\sum_{t=\tau}^{T+\tau}\left[(\delta_n - \delta_o)\times(P_{n,t} - P_{n,t-1})\right]|\delta_n\right\},$$

where the expectation, $E\{\cdot|\delta_n\}$, is based upon the true process, and where the market's conditional forecasts, $E_{t-1}(s_t|P_{j,t-1})$, are based upon the $t-1$ information set of (z_t, n_t) and upon learning about the processes up until time $t-1$, as embodied in the conditional probabilities.

From equation (10), we can clearly see that the expected value of a sample mean of

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[4] The result is straightforward and is discussed in Karen Lewis (1988a).

forecast errors based upon realizations of z_t from the true distribution, δ_n, is negative. If the process has in fact changed, then the first component $\hat{v}_{n,t} \equiv v_{n,t}$, which has mean zero. However, the estimate of the disturbance from the "Old" process based upon realizations from the "New" process is on-average positive since it is given by:

$$\hat{v}_{o,t} = \Delta z_t - \delta_o = (\delta_n + v_{n,t}) - \delta_o, \quad \text{for } \delta = \delta_n. \tag{11}$$

Intuitively, during the learning period, too much of the larger observed fundamental variable is associated with transitory noise relative to a permanent change. Thus, the first component of forecast errors given by $P_{o,t}(v_{o,t} - v_{n,t})$ will be positive for an average sequence of drawings from the true distribution. As $P_{o,t}$ goes to zero, this component converges to zero as well.

The second source of on-average mis-prediction arises from the expected permanent growth rate of the fundamental. Because the market does not initially believe with certainty that the process has changed, $P_{n,\tau} < 1$. Therefore, as $P_{n,t}$ rises during the learning period, the sum of the change in probabilities, $P_{n,t} - P_{n,t-1}$, is positive in expectation. Intuitively, while learning the market does not yet fully believe that the process is δ_n and therefore underestimates on-average the fundamental's permanent growth rate. For this reason, the expected value of the second component, $-(1+\alpha)(\delta_n - \delta_o)(P_{n,t} - P_{n,t-1})$ based upon the true δ_n is negative during the learning period.

Since the market's average mis-prediction disappears as $P_{o,t}$ goes to zero, it might seem that faster learning will always imply less mis-prediction on-average. This intuition is misleading, however. Within any small sample, the speed of convergence in the probabilities affects the two components of the "bias" in equation (10) in opposite directions. For example, very slow downward movement in $P_{o,t}$ increases the bias due to the transitory component, $e_{1,t}$, but because changes in $P_{n,t}$ are smaller, it also reduces the bias due to the permanent component, $e_{2,t}$; and vice versa for relatively fast probability convergence. Both of these cases appear in the results examined below.

II. Empirical Evidence Using U.S. Money Demand

As described in the introduction, survey data and forward exchange rates suggest that the market was systematically surprised by the strength of the U.S. dollar during the early 1980s, in apparent contradiction to the premise of rational expectations. However, the preceding discussion demonstrated that on-average systematic forecast errors could arise from rational behavior if the market were learning about a shift in the process of fundamentals. Relating this theoretical discussion to the foreign exchange market requires identifying relevant exchange rate "fundamentals."

Motivations for the appropriate fundamentals variables that influence the exchange rate range from trade balance effects (Peter Hooper and John Morton, 1982) to fiscal policy (Martin Feldstein, 1986) to international price adjustment (Michael Mussa, 1982), Maurice Obstfeld and Kenneth Rogoff, 1984), to name only a few. While these fundamental effects may be important, learning behavior applied to a particular one of these fundamentals would be model-specific. On the other hand, money market equilibrium is a required condition common to many different exchange rate models and therefore is the focus of this section.

A. *The U.S. Money Market in the Early 1980s*

Beginning in 1981, money balances substantially exceeded most projections based upon money demand equations then in use by various sources including the Federal Reserve, leading some to call the episode the "Great Velocity Decline" (for example, Federal Reserve Bank of San Francisco, 1983). Indeed, despite a fairly stable and positive annual growth rate for $M1$ velocity of about 3.4 percent from 1947 to 1981, the rate of velocity growth was *negative* from the fall of

1981 through 1986. Also, in terms of money demand itself, a number of studies have identified a positive shift in U.S. money demand around the fall of 1981.[5]

If fully accommodated by an increase in the money supply, this shift in money demand growth would not affect the exchange rate. However, there are two main reasons to believe that the increase in money demand was not immediately offset by increased money supply. First, at the time of the shift, the Federal Reserve was conducting monetary policy using a non-borrowed reserves target, an operating procedure that does not fully accommodate changes in money demand. The increased growth rate in monetary aggregates following the apparent increase in money demand eventually helped induce the Federal Reserve to abandon the non-borrowed reserves target in the summer to autumn of 1982. The operating procedure was officially replaced with a borrowed reserves target in early 1983 together with a more judgmental approach to targeting that has again implied partial, but not complete, monetary accommodation.[6]

The second main reason for incomplete monetary accommodation is that the Federal Reserve appeared to use money demand projections to set non-borrowed reserves targets at FOMC meetings for the six- to eight-week inter-meeting period (see, for example, David Lindsey, 1981). Hence, even an overt decision to accommodate the increased money demand as a matter of discretionary policy would have required enough observations for the Fed to adjust its projections used in forming policy. Furthermore, such an overt decision seems unlikely since long-term targets were apparently taken seriously and readjusted only infrequently (Richard Davis, 1981).

Thus, given the nature of operating procedures and the policymaking process by the Federal Reserve during the early 1980s, an increase in money demand would probably have taken time to accommodate. An increase in money demand without a commensurate increase in money supply would have induced an appreciation of the U.S. dollar. For this reason, the following analysis will treat the increase in money demand as a source of shift in fundamentals. However, offsetting increases in the money supply could in theory mitigate the implied appreciation of the dollar due to the increased money demand. Because of this possibility, parameter values that minimize the impact of exchange rate mis-predictions will be emphasized in the calculations of learning effects below.

B. *Forecast Errors and a Shift in Domestic Money Demand*

To investigate empirically the market's assessed probabilities of the new money market process as specified in equation (9), we need a money demand equation that is parsimonious. We require parsimony since a single update of the probabilities necessitates enough independent observations of the fundamentals process to identify the model. That is, if k is the number of parameters in the money demand equation, the probabilities can be revised only every k periods. For this reason, the following form of the money demand equation was used in the analysis.

$$(12)\quad \Delta m_t - \Delta p_t = \delta_j - \theta \Delta i_t + v_{j,t}, \qquad j = o, n,$$

where m and p are the logarithms of domes-

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[5] Reasons posited for this shift range from the dramatic decline in inflation, to a portfolio switch out of bonds due to the increased volatility in interest rates, to a combination of effects from financial innovation in conjunction with the decline in inflation. On the behavior of velocity, see Robert Heller (1988). On the behavior of different money demand equations see, for instance, Yoshihisa Baba, David Hendry, and Ross Starr (1988), Andrew Rose (1985), and the references therein.

[6] See Robert Heller (1988) for a description of how non-borrowed reserves targeting implies only partial accommodation. As he demonstrates, total reserves have fluctuated more under the recent borrowed reserves targeting procedure than under the period of non-borrowed reserves targeting, but interest rates have also fluctuated more than under the period of Fed funds targeting in the 1970s, again suggesting partial accommodation. Partial accommodation of the shifts in velocity during the early 1980s has been argued by Olivier Blanchard (1984) and Karl Brunner and Allan Meltzer (1983), among others.

tic money and the price level, respectively, i_t is the level of the domestic interest rate, δ is a constant term, θ is the interest semi-elasticity of money demand, and v is a normally distributed i.i.d. disturbance term. Two assumptions are embodied in equation (12) for the sake of parsimony. First, the income elasticity is constrained to zero, although probability estimates based upon setting the income elasticity at other levels did not substantially alter the results.[7] Second, the disturbance to the money demand equation in level terms is assumed to contain a unit root, with a white-noise innovation after first-differencing. This specification is consistent with the form of money demand assumed in empirical specifications of the monetary model of exchange rate determination such as Richard Meese (1986) and Kenneth West (1987).

In this form, U.S. money demand may be viewed as the fundamental variable represented by z_t in equation (1).[8] In other words,[9]

$$(13)\quad \begin{aligned} \Delta z_t &\equiv \Delta m_t - \Delta p_t - \theta \Delta i_t \\ &= \delta_j + v_{j,t}, \qquad\qquad j = \text{o}, n. \end{aligned}$$

When δ increases to $\delta_n > \delta_o$, unless the market participants immediately recognize this change, they on-average underestimate the strength of the domestic currency while they learn that z_t follows the new process.

To verify the shift in money demand found in other studies, the constancy of the parameters in the money demand equation (12) was tested. The monthly money and price data are from Richard Meese (1986), covering the period from January 1973 to June 1984 and are $M1$ and CPI data, respectively. The interest rates are from Morgan Guaranty's World Financial Markets. Indeed, using a Wald test, the constancy of δ before and after October 1981 was rejected at a marginal significance level of 0.02 percent, consistent with the shift found using other forms of money demand.

C. *Constructing the Forecast Errors*

Given this shift in money demand, we might ask how predictions about the dollar would have been affected if the market were learning about the change. One way to gauge the impact of learning upon the exchange rate is to consider the implied effects based upon some extreme assumptions about the learning process. Therefore, this section calculates effects from learning under one extreme assumption: the market knows the parameters of the new distribution; while the next section assumes the other extreme: the market has no information about the new distribution. Presumably, the "true" case is bounded between these two extremes.

Calculating the effects of learning on the *ex post* average mis-prediction described by the variable e_t requires three sets of variables. The first set of variables, δ_j and $\hat{v}_{i,t}$, are estimated from the data using the money demand equation (12). The second set, variable α, corresponds to the characteristic root of the full exchange rate model and will be discussed in more detail below. The third set of variables are the probabilities, $P_{j,t}$, that determine the evolution and convergence of the dollar's systematic mis-prediction. As described in equation (9), these probabilities depend upon a prior probability. Rather than specifying an *ad hoc* prior probability, how-

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[7] The income elasticities investigated were 0.4 and 0.3, values Richard Meese (1986) reports unrejected by a monetary model of the dollar.

[8] This money demand is assumed to be the market aggregate of a very large number of atomistic agents. Although individual agents have information about their own money demand, they view their contribution as having no effect upon the aggregate. Therefore, they learn about aggregate money demand by observing the market.

Notice also that focusing upon U.S. money demand alone treats foreign money demand as one of the "other fundamentals" in N_t. However, since monetary models typically depend upon the *difference* between domestic and foreign money demand, the following learning analysis was also applied to the United States minus German and United States minus British money demand functions. As reported in Karen Lewis (1988b), the implied forecast errors using relative money demand are similar to those using U.S. money demand alone.

[9] In standard monetary models of the exchange rate such as Michael Mussa (1976), the characteristic root of the exchange rate solution is generally a function of the interest semi-elasticity, θ. For this reason, the interest rate response of money demand does not enter directly as fundamentals.

ever, we can provide an estimate of this initial probability by assuming that the market has essentially learned the new process by some reasonable endpoint. At this point, we can specify a terminal "new" probability close to one. Then we can "back out" the probabilities by taking the posterior odds of equation (9) and moving these odds backward through time according to:

$$(14)\quad \left(\frac{P_{n,t-k}}{P_{\mathrm{o},t-k}}\right)=\frac{P_{n,t}f(\Delta z_t,\dots,\Delta z_{t-k}|\delta_{\mathrm{o}})}{P_{\mathrm{o},t}f(\Delta z_t,\dots,\Delta z_{t-k}|\delta_n)}.$$

Since $f(\cdot)$ is the money demand equation (12), the minimum number of observations the market requires in order to identify the process equals the number of parameters (i.e., δ,θ,σ_v), so that $k=3$.

Since the econometrician typically has less information available than the market has, we may benchmark the latest feasible terminal period by noting endpoints of sample periods used by academic studies that note the apparent money demand change. By this criterion, an outer bound for learning convergence of July 1984 was chosen.[10] Although the first set of probabilities will be backed out from this point, the probabilities are recursive functions only of the likelihood ratios so that choosing a probability at any point in time and iterating equation (9) forward and backward determines a unique path of probabilities.[11]

In addition to choosing an initial probability, calculating the probabilities requires forming the likelihood ratio of the two money demand distributions for each observation. The parameters of the "old" distribution, denoted δ_0, was estimated using data during the floating rate period from July 1973 through September 1981. Similarly, the "new" distribution, denoted δ_n, was estimated from October 1981 through June 1984. For the analysis presented below, the disturbance variance, σ_v, was assumed the same over the two processes although allowing for different variances did not appreciably alter the results. Karen Lewis (1988b) details the construction of these probability estimates.

D. *Empirical Evidence: Evolving Beliefs About U.S. Money Demand*

Table 1 presents the evolution of the probabilities for the new higher U.S. money demand equation given that the probabilities have almost converged by mid-1984 with two assumed final probabilities of the old process: 0.1 percent and 1 percent. The probabilities were then "backed out" to the end of 1981, as described in equation (14). The columns with headings P_n describe the behavior of the "new" probability over time. During much of 1982, the market does not yet have enough information to assess whether the money demand equation has changed to δ_n. But over time, the market begins to recognize that money demand is governed by the new equation so that the probabilities of the new process converge. The results in the table also indicate that backing out the probabilities implies very small initial probabilities of less than 1 percent, estimates that may seem reasonable since the change appeared largely unanticipated. However, since the recursive probabilities depend only upon the likelihood ratio, we can also consider the effects of larger initial probabilities from the Table 1 results. The table clearly indicates that higher initial probabilities of P_n would imply even larger final probabilities of the new distribution than 0.999. By contrast, the learning model in the following section indicates a wider feasible range of initial probabilities.

The evolution of this probability affects the behavior of e_t, the degree of *ex post* "bias" in forecast errors during the fall of 1981 through mid-1984.[12] This behavior de-

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[10] For example, Baba, Hendry, and Starr (1987) use a data set ending in the second quarter of 1984 to help explain the "great velocity decline" with a money demand specification.

[11] Therefore, readers who may believe that the market learned about the change in money demand after July 1984 would choose a relatively large terminal value of P_{o}. However, terminal values greater than about 1 percent imply almost implausibly small initial probabilities for the new process, as will be shown below. In addition, the learning analysis begins in October 1981 since before this time, the probabilities in equation (14) would fall on-average.

[12] Recently, Charles Engel and James Hamilton (1988) and Graciela Kaminsky (1988) have estimated

pends upon the size of the increase in money demand, given by δ_0 and δ_n at the top of the table. In addition, calculating $e_{2,t}$, the permanent component, requires a value of α. In general, this parameter determines the characteristic root of the exchange rate equation and therefore depends upon the full exchange rate model, potentially including the dynamics of the omitted variables, N_t.[13] To understand the impact of α, observe from equation (10) that the absolute value of the "bias" due to the permanent money demand component depends positively upon α. As equation (2) shows, larger values of α imply that future expectations have a stronger effect upon the current exchange rate and, therefore, larger effects upon exchange rate forecast errors.

Since larger values of α bias the learning effects upward, Table 1 reports values of the *ex post* bias terms for two "lower-bound" values of α discussed in Behzad Diba (1987). He explains why some exchange rate studies that assume a lower-bound level of $\alpha = 0.8$ choose a range of α that is too low since they do not adjust for the difference between annual and monthly data. He suggests that instead $\alpha = 100$, but also finds that lower-bound estimates of 14 give implied exchange rate variances at least as large as actual exchange rate variances. To allow comparison with this literature, Table 1 reports results assuming these two lower-bound estimates: $\alpha = 0.8$ and 14.

Several issues concerning the behavior of these forecast errors in Table 1 deserve emphasizing. First, as described in Section I, the size of the *ex post*, apparent bias disappears over time as the probability converges. For example, the effects of the disturbance term component, $P_0(v_0 - v_n)$, reaches a peak in September 1982 and generally declines thereafter, dissipating to small levels by the end of the sample. Second, the probabilities are random variables, evident from the variability in the component under the columns marked "$e_{2,t}$." Third, as demonstrated at the bottom of the table, the mean of the forecast errors implied by changing beliefs about U.S. money demand are about 0.7 when $\alpha = 14$ but decline to between 0.4 to 0.5 when $\alpha = 0.8$. Overall, these lower values correspond to roughly a half of the systematic underprediction of the dollar based upon the forward markets in the German DM and the British pound.[14]

Since e_t is a random variable, this learning process also implies greater variability in forecast errors. Although one might suppose that testing whether the variability of e_t is significantly related to the variability in the exchange rate would comprise a test of the model, two factors preclude such an interpretation. First, the learning process is inherently a small sample problem and therefore asymptotic properties do not apply. The second and less obvious reason arises because by construction e_t is a variable that closely converges within the sample to its asymptotic distribution of zero (with no variance). Since a covariance with any nonrandom constant is zero, measures of the covariance between the mis-prediction term, e_t, and the exchange rate will be biased toward zero. We might nevertheless inspect these covariances for different parameter values as a general indication of the behavior of the model. As Table 1 reports, the covariances between the forward prediction errors and the implied errors are positive in all 4 cases—for the two different terminal probabilities and for the range of α.[15]

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time-series processes of the exchange rate that parameterize two different regimes of appreciation and depreciation, respectively. A challenge for future research will be to understand the combination of changes in fundamentals behind these switches in exchange rate regimes. The period under study above falls within one of their dollar appreciation regimes of roughly March 1981 to February 1985 and therefore makes a contribution toward relating this "regime" to a change in a fundamental equilibrium condition.

[13] For example, Richard Meese and Kenneth Singleton (1983) solve for exchange rate variance bounds relationships using general monetary models as well as the two-good model of Michael Mussa (1982), in which the roots of the exchange rate solution depend upon the dynamic behavior of prices.

[14] The exchange rates are from the IMF's "International Statistics Monthly" while the forward rates are constructed from the interest rate data assuming covered-interest parity.

[15] Although positive covariances were also found for other terminal probabilities, by using different values

TABLE 1—IMPLIED EXCHANGE RATE FORECAST ERRORS USING LOWER-BOUND ESTIMATES OF α EVOLUTION AND SUMMARY STATISTICS OF THE PROBABILITIES AND THE FORECAST ERRORS

Ex Post Forecast Bias: $e_{1,t}+e_{2,t}\equiv P_o(v_n-v_o)+(1+\alpha)\Delta P_n(\delta_n-\delta_o)$

$\delta_o=-0.303,\ \delta_n=0.362$

Final Probability of Old Process	0.001				0.010			
	P_N	$e_{1,t}$	$e_{2,t}$ $\alpha=0.8$	$\alpha=14$	P_N	$e_{1,t}$	$e_{2,t}$ $\alpha=0.8$	$\alpha=14$
Month								
82:01	0.000	0.96	0.000	0.000	0.000	0.96	0.000	0.000
82:02	0.002	0.54	0.002	0.018	0.000	0.55	0.000	0.002
82:03	0.000	0.56	−0.002	−0.017	0.000	0.56	−0.002	−0.002
82:04	0.000	0.68	0.000	0.000	0.000	0.68	0.000	0.000
82:05	0.002	0.64	0.002	0.018	0.000	0.64	0.000	0.002
82:06	0.000	0.67	−0.002	−0.016	0.000	0.67	−0.000	−0.002
82:07	0.000	0.72	−0.000	−0.001	0.000	0.72	−0.000	−0.000
82:08	0.005	0.53	0.007	0.054	0.000	0.53	0.000	0.005
82:09	0.009	0.92	0.004	0.033	0.001	0.93	0.000	0.003
82:10	0.000	0.86	−0.010	−0.085	0.000	0.86	−0.001	−0.008
82:11	0.036	0.59	0.042	0.353	0.004	0.61	0.004	0.036
82:12	0.079	0.73	0.052	0.433	0.008	0.79	0.006	0.048
83:01	0.026	0.65	−0.063	−0.528	0.003	0.66	−0.007	−0.058
83:02	0.550	0.29	0.628	5.231	0.108	0.58	0.126	1.053
83:03	0.506	0.35	−0.053	−0.442	0.092	0.64	−0.019	−0.160
83:04	0.112	0.59	−0.472	−3.932	0.012	0.66	−0.096	−0.797
83:05	0.437	0.33	0.389	3.239	0.071	0.54	0.071	0.588
83:06	0.319	0.50	−0.141	−1.175	0.044	0.70	−0.032	−0.269
83:07	0.732	0.18	0.495	4.121	0.213	0.52	0.202	1.682
83:08	0.966	0.02	0.281	2.338	0.739	0.16	0.630	5.251
83:09	0.978	0.01	0.014	0.120	0.817	0.12	0.093	0.776
83:10	0.843	0.11	−0.161	−1.346	0.348	0.44	−0.561	−4.679
83:11	0.979	0.01	0.163	1.356	0.824	0.12	0.570	4.748
83:12	0.989	0.01	0.012	0.098	0.900	0.07	0.091	0.755
84:01	0.979	0.01	−0.012	−0.097	0.824	0.11	−0.090	−0.753
84:02	0.994	0.00	0.018	0.149	0.945	0.04	0.144	1.204
84:03	0.996	0.00	0.002	0.016	0.959	0.03	0.017	0.144
84:04	0.993	0.01	−0.004	−0.032	0.930	0.04	−0.034	−0.287
84:05	0.990	0.01	−0.003	−0.023	0.911	0.06	−0.024	−0.197
84:06	0.992	0.01	0.002	0.013	0.922	0.05	0.014	0.116

Forward Prediction Error Means: German DM = −0.95 British Pound = −1.08

Forward Prediction Error Variances: German DM = 8.05 British Pound = 6.56

Implied Errors:	Mean	Variance	Covariance w/Forward Error German DM	British Pound
$\alpha=14$/Final P_0 of 0.001	−0.71	2.57	0.27	0.14
$\alpha=14$/Final P_0 of 0.010	−0.77	2.50	0.40	0.06
$\alpha=0.8$/Final P_0 of 0.001	−0.42	0.12	0.04	0.04
$\alpha=0.8$/Final P_0 of 0.010	−0.50	0.10	0.07	0.01

Notes: [a]The forecast error estimates use U.S. *M*1 money supply, CPI, and industrial production data described in Richard Meese and Kenneth Rogoff (1984). Interest rates are from Morgan Guaranty's *World Financial Markets*, while exchange rates are from the IMF's *International Statistics Monthly*. Probability estimates are based upon normal conjugate prior distributions with $\delta_o=-0.303$, $\delta_n=0.362$, Precision parameters: $\bar{s}^2=4.84$, $q=135$.

Larger values of α than assumed in Table 1 will clearly imply greater average exchange rate mis-prediction over the period. Therefore, a useful criterion for determining the range of α consistent with the model is to ask: for what value of α would learning explain *all* of the observed under-prediction? Table 2 reports the forecast error series calculated by choosing critical levels of $\bar{\alpha}$

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for the probabilities to generate forecast errors it was possible to generate negative covariances in some cases.

TABLE 2—IMPLIED EXCHANGE RATE FORECAST ERRORS USING BREAK-EVEN VALUES FOR α EVOLUTION AND SUMMARY STATISTICS OF THE FORECAST ERRORS

Ex Post Forecast Bias: $.95 = e_t = e_{1,t} + e_{2,t} = P_o(v_n - v_o) + (1+\alpha)\Delta P_n(\delta_n - \delta_o)$

$\delta_o = -0.303$, $\delta_n = 0.362$

Implied Forecast Errors

Final Probability of Old Process[a]	0.001		0.010	
Critical $\bar{\alpha}$	25.81		23.61	
	Total Error	Permanent Component	Total Error	Permanent Component
	e_t	$e_{2,t}$	e_t	$e_{2,t}$
Month				
82:01	0.959	0.000	0.959	0.000
82:02	0.574	0.030	0.548	0.003
82:03	0.527	−0.030	0.554	−0.003
82:04	0.678	0.000	0.678	0.000
82:05	0.672	0.031	0.645	0.003
82:06	0.637	−0.028	0.663	−0.003
82:07	0.717	−0.002	0.719	−0.000
82:08	0.624	0.093	0.542	0.008
82:09	0.976	0.057	0.932	0.005
82:10	0.712	−0.146	0.845	−0.013
82:11	1.198	0.607	0.667	0.057
82:12	1.476	0.745	0.863	0.075
83:01	−0.260	−0.908	0.573	−0.091
83:02	9.295	9.001	2.241	1.658
83:03	−0.411	−0.760	0.391	−0.251
83:04	−6.177	−6.767	−0.598	−1.254
83:05	5.900	5.573	1.465	0.926
83:06	−1.525	−2.022	0.273	−0.424
83:07	7.268	7.091	3.167	2.648
83:08	4.044	4.024	8.425	8.266
83:09	0.221	0.207	1.339	1.221
83:10	−2.211	−2.316	−6.929	−7.365
83:11	2.347	2.333	7.595	7.475
83:12	0.176	0.168	1.257	1.189
84:01	−0.154	−0.168	−1.069	−1.185
84:02	0.260	0.257	1.930	1.189
84:03	0.030	0.027	0.256	0.227
84:04	−0.050	−0.054	−0.407	−0.452
84:05	−0.033	−0.039	−0.255	−0.309
84:06	0.029	0.023	0.233	0.183

Forward Prediction Error Means: German DM = −0.95 British Pound = −1.08

Forward Prediction Error Variances: German DM = 8.05 British Pound = 6.56

Implied Errors:	Mean[b]	Variance	Covariance w/Forward Error German DM	Covariance w/Forward Error British Pound
Final P_0 of 0.001	−0.95	7.70	0.46	0.23
Final P_0 of 0.010	−0.95	6.30	0.62	0.10

Notes: [a] The data used and the evolution of the probabilities are the same as in Table 1.
[b] By construction.

that set the sample average of the implied forecast error series equal to the sample average forward prediction error of the dollar-deutsche mark exchange rate, 0.95. Since the evidence in Jeffrey Frankel and Ken Froot (1987) indicate that forecast errors based upon survey data were generally larger than the forward prediction errors, this estimate of the prediction error might even be considered relatively small. The critical values implied by forcing the model to explain all of the *ex post* bias in forecasting are 25.81 and 23.61. As the table shows, both forecast error series have more variability and are positively correlated with forward prediction errors in all cases.

Overall, the results in this section indicate that a relatively wide range of α yield a range of the under-prediction of the dollar's strength from about one-half to all of the forward market's under-prediction during the period from 1981 to 1984. Within the sample, the *ex post* bias due to the permanent component of money demand, $e_{2,t}$, tended to be somewhat large since the probabilities converged rather quickly. This relatively fast convergence of the probabilities depended in part upon the underlying assumption that the market knew the parameters of the new and old distributions.

III. Evolving Beliefs While Learning the Process Parameters

By contrast with the previous discussion, this section investigates the effects upon dollar exchange rate forecast errors assuming the market did not know the new distribution of money demand but instead learned its parameters over time.[16] In this version, the market learns the distribution by updating for each observation of z_t its priors of the "old" and "new" parameter distributions, δ_o and δ_n, respectively. Defining as $\hat{\delta}_{i,t}$ the parameter estimate formed from the posterior distribution under process i, the exchange rate equation (7) requires the following modifications:

$$(7') \quad s_t = -[P_{o,t}\hat{v}_{o,t} + P_{n,t}\hat{v}_{n,t}] - (1+\alpha)[P_{o,t}\hat{\delta}_{o,t} + P_{n,t}\hat{\delta}_{n,t}] - z_{t-1} + {}_tN_t.$$

Since market participants initially have no information about δ_n, they use a diffuse prior for its distribution at τ. Using subsequent observations of z_t, they update this prior distribution providing new estimates of $\hat{\delta}_{n,t}$. On the other hand, they base their prior distribution of the old process, δ_o, at τ upon the past history of z_t and use observations following τ to update this prior.[17]

Taking forecast errors conditional upon the two prior distributions, parameterized by $\hat{\delta}_j$ and upon the prior probabilities parameterized by $P_{j,t}$ implies,

$$(8') \quad \left(s_t - E_{t-1}\left(s_t | \hat{\delta}_{j,t-1}, P_{j,t-1}\right)\right) = -\hat{v}_{n,t} - P_{o,t}(\hat{v}_{o,t} - \hat{v}_{n,t}) - (1+\alpha) \times \left[\left(P_{o,t}\hat{\delta}_{o,t} - P_{o,t-1}\hat{\delta}_{o,t-1}\right) + \left(P_{n,t}\hat{\delta}_{n,t} - P_{n,t-1}\hat{\delta}_{n,t-1}\right)\right] = -\hat{v}_{n,t} - \tilde{e}_{1,t} - \tilde{e}_{2,t},$$

where now $\hat{v}_{n,t} = \Delta z_t - \hat{\delta}_{n,t}$ and $\tilde{e}_{n,t} = P_{o,t} \times (\hat{v}_{o,t} - \hat{v}_{n,t})$. Thus, even though the forecast errors are more complicated under parameter learning, the basic results from the simpler model continue to apply. As before, if the process in fact changed at τ, the disturbances based upon the "new" distribution, $\hat{v}_{n,t}$, have expectation zero since the expected value of $\hat{\delta}_{n,t}$ is δ_n, the true mean of the posterior distribution.[18] Also as before, the expected value of the disturbance based upon the "old" process, $\hat{v}_{o,t}$, is positive.[19] Therefore, while market participants are learning about the new process, they ascribe too much of the money demand observation to transitory noise by the weight placed upon the old process, $P_{o,t}$. Finally as before, the second component in (8′), $\tilde{e}_{2,t}$, arises from under-

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[16] Examples of papers that study the effects of learning about the market parameters include Roman Frydman (1982), Margaret Bray and N. E. Savin (1986), and Albert Marcet and Tom Sargent (1986) for self-referential learning.

[17] See Lewis (1989) for a discussion of the evolution of both of these distributions.

[18] See Arnold Zellner (1971), pp. 224–33.

[19] To see this result, note that the disturbance term conditional upon δ_n is: $\hat{v}_{o,t} = \Delta z_t - \hat{\delta}_{o,t} = \delta_n + v_{n,t} - \hat{\delta}_{o,t}$. Since initially $\hat{\delta}_{o,\tau} < \delta_n$, for small samples, $\hat{v}_{o,t}$ has positive expectation based upon the true distribution, δ_n.

predicting the permanent growth rate of money demand during learning. The expectation of $\tilde{e}_{2,t}$ based upon the true distribution is positive because as $P_{n,t}$ is increasing, the estimates of $\hat{\delta}_{n,t}$ are rising faster than $\hat{\delta}_{o,t}$.

The forecast errors in equation (8) were calculated based upon initial prior probabilities and distributions for δ in October 1981. The prior distribution for the "old" process was estimated from equation (12) using data during the floating rate period since 1973, while the initial "new" prior was diffuse. Since the market initially has no information about the new distribution, the market learns much more slowly than when the parameters are known. For this reason, backing out the probabilities as in equation (14) implied implausibly large initial probabilities of a change in money demand.[20] Given this evidence, we may proceed to consider the effects upon the forecast errors based upon a range of initial probabilities as presented in Table 3. The top of the table reports some summary statistics on the behavior of the parameter estimates under the "Old" and "New" beliefs about the money demand process. First, the average parameter estimates for δ_o and δ_n were -0.16 and 0.21, respectively. Although the parameter estimates evolve over time, the average value of $\hat{\delta}_{o,t}$ is less than the average value of $\hat{\delta}_{n,t}$ since the market weights observations before 1981 in the estimate of money demand in forming $\hat{\delta}_{o,t}$. Also, as the summary evidence demonstrates, the variance of the parameter based upon believing a change has occurred, $\hat{\delta}_{n,t}$, is much larger than the "no-change" parameter estimates, $\hat{\delta}_{o,t}$.

To gauge the sensitivity of the implied forecast errors to initial probabilities, Table 3 reports summary statistics of probabilities and implied forecast errors for two very different initial probabilities of a "New" process: (1) $P_{n,\tau} = 50$ percent indicating a market that thought a change to a new process was equally likely as no change, and (2) $P_{n,\tau} = 1$ percent indicating a market with low initial beliefs of a change. The table reports the results of the lower-bound case where $\alpha = 0.8$, although the bottom of the table contains summary statistics assuming $\alpha = 14$ as well. The probability of a change generally rises over time although neither probability process converges to one within the sample. Since the probabilities converge much more slowly in this case, the permanent component, $\tilde{e}_{2,t}$, exhibits less mis-prediction. However, as the summary statistics indicate, both series imply negative average forecast errors consistent with the lower-bound range found in the previous tables. Furthermore, the variability is larger than before since the market now learns about the parameter estimates in addition to detecting the process change.

The table also reports the results using critical levels of $\bar{\alpha}$ that set the implied error means equal to the dollar-DM forward error mean of 0.95. The critical values of $\bar{\alpha}$ comprise a rather wide range from 12.23 to 84.83 for initial "new" probabilities of 0.5 and 0.01, respectively. The initial probability of 0.5 implies excessively large variances, however, indicating that, if the market were learning in this manner, either the value of α or the initial probability of 50 percent are too high. The correlation between the implied errors and the British pound are positive in all cases and relatively large, but the correlation with the German DM are essentially zero or negative.

Overall, the results assuming that the market learned about the new distribution of money demand indicates that the market recognized the shift in the fundamental variable much more slowly. In contrast to the known parameter version of learning, this slower learning increases the implied underprediction due to the transitory component of money demand but reduces that due to the permanent component. However, the weak correlation between the model and the DM forward prediction errors suggests that actual learning was more likely based upon a prior for δ_n with more information than in a diffuse prior.

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[20] For instance, given a terminal probability of "Old" equal to 0.05 implied an initial probability of a "New" money demand distribution of 0.86 in the final quarter of 1981.

TABLE 3—IMPLIED EXCHANGE RATE FORECAST ERRORS WITH PARAMETER LEARNING EVOLUTION AND SUMMARY STATISTICS OF THE PROBABILITIES AND THE FORECAST ERRORS

Ex Post Forecast Bias: $e_t \equiv P_0(\hat{v}_{n,t} - \hat{v}_{0,t}) + (1+\alpha)[(P_{0,t}\hat{\delta}_{0,t} + P_{n,t}\hat{\delta}_{n,t}) - (P_{0,t-1}\hat{\delta}_{0,t-1} + P_{n,t-1}\hat{\delta}_{n,t-1})]$

δ_0: Average -0.160, Range $-0.26/-0.09$, Variance 0.003

δ_n: Average 0.206, Range $-0.46/1.44$, Variance 0.111

For Initial Probability of New Process		0.5		0.01		
	P_N	e_t $\alpha = 0.8$	e_t $\bar{\alpha} = 12.2$	P_N	e_t $\alpha = 0.8$	e_t $\bar{\alpha} = 84.8$
Month						
81:12	0.50	5.33	28.63	0.010	3.38	7.60
82:01	0.50	−1.41	−9.90	0.010	−0.24	−4.23
82:02	0.11	−0.35	−2.10	0.001	−0.12	−1.64
82:03	0.41	0.33	1.88	0.007	0.19	1.79
82:04	0.49	0.44	1.52	0.007	0.47	1.88
82:05	0.07	0.14	−1.59	0.001	0.44	0.33
82:06	0.69	−0.05	0.52	0.021	−0.47	−2.32
82:07	0.41	0.76	−0.36	0.007	1.58	2.29
82:08	0.16	0.93	1.26	0.002	1.07	2.97
82:09	0.18	−0.09	−0.71	0.030	−0.35	−1.19
82:10	0.87	0.25	1.35	0.650	0.58	3.29
82:11	0.25	0.36	1.27	0.003	0.34	2.91
82:12	0.83	0.64	4.20	0.048	0.50	3.85
83:01	0.88	0.03	0.02	0.071	0.31	1.33
83:02	0.23	−0.34	−3.87	0.003	0.20	−3.37
83:03	0.70	0.61	3.47	0.023	0.57	2.47
83:04	0.89	0.31	2.03	0.075	0.39	4.40
83:05	0.25	−0.31	−4.23	0.003	0.32	−3.51
83:06	0.78	0.64	3.89	0.035	0.63	2.81
83:07	0.84	0.19	0.84	0.051	0.55	1.93
83:08	0.17	−0.27	−4.29	0.002	0.38	−2.23
83:09	0.65	0.50	2.79	0.018	0.41	1.07
83:10	0.76	0.21	0.92	0.030	0.43	1.50
83:11	0.39	0.04	−1.53	0.006	0.47	0.46
83:12	0.76	0.57	3.23	0.031	0.67	3.05
84:01	0.72	−0.01	−0.80	0.025	0.36	−0.63
84:02	0.32	−0.16	−2.90	0.005	0.34	−2.30
84:03	0.66	0.45	2.25	0.019	0.50	1.61
84:04	0.71	0.28	1.17	0.024	0.53	2.30
84:05	0.36	0.05	−1.84	0.006	0.52	−0.29
84:06	0.73	0.46	2.35	0.026	0.58	1.32

Forward Prediction Errors:	Mean	Minimum	Maximum	Variance
German Deutsche Mark	0.95	−5.06	7.72	8.05
British Pound	−1.08	−5.03	6.09	6.56

Implied Errors:	Mean	Variance	Covariance w/Forward Errors	
			German DM	British Pound
$\alpha = 0.8$/Initial P_n of 0.50	−0.34	1.06	−0.04	0.39
$\alpha = 0.8$/Initial P_n of 0.01	−0.50	0.43	−0.04	0.08
$\alpha = 14$/Initial P_n of 0.50	−1.05	44.79	0.00	3.41
$\alpha = 14$/Initial P_n of 0.01	−0.57	0.79	−0.12	0.22
$\bar{\alpha} = 12.23$/Initial P_n of 0.50	−0.95[a]	35.16	0.00	3.01
$\bar{\alpha} = 84.83$/Initial P_n of 0.01	−0.95[a]	6.93	−0.52	0.96

Notes: [a]The forecast error estimates use U.S. *M*1 money supply, CPI, and industrial production data described in Richard Meese and Kenneth Rogoff (1984). Interest rates are from Morgan Guaranty's *World Financial Markets*, while exchange rates are from the IMF's *International Statistics Monthly*.

IV. Concluding Remarks

This paper investigated the effects upon average dollar forecast errors following the increase in U.S. money demand in the early 1980s as the market was learning about the new process of money. For relatively conservative parameter values, the magnitude of under-prediction of the dollar's strength appeared to correspond to roughly one-half of the under-prediction implied by the forward exchange rate during the same period. Although this analysis represents a useful initial investigation into the effects of revising beliefs about the fundamentals process, a noteworthy issue remains. Contrary to the implications of this once-and-for-all switch in fundamentals with learning, the systematic nature of the prediction errors implied by the forward rate in the foreign exchange market or by survey data do not appear to die out over time. Although the systematic nature of forecast errors may appear more pronounced over some time intervals, the persistence in this behavior over longer periods implies that learning about a change in fundamentals cannot be the only explanation. Thus, the apparent systematic behavior of prediction errors over longer time periods may arise from a combination of learning behavior together with anticipations of future policy changes and risk premia.

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