

Occasional Interventions to Target Rates

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This paper develops a model of central-bank intervention based upon a policy characteristic of foreign-exchange interventions by the United States, Germany, and Japan in the late 1980's and evaluates it empirically. Central bankers intervene with greater intensity as rates deviate from target levels, but they also try to stabilize rates around current levels. The model is estimated using exchange rates and data based upon observed central-bank interventions. Interestingly, the estimates of the model are consistent with the predictions of the theoretical model for both the deutsche-mark / dollar rate and, less strongly, for the yen / dollar rate. (JEL F41, G15, F31)

Governments frequently target macroeconomic variables through a mixed policy of occasional interventions with otherwise floating rates. This type of policy has been particularly characteristic of foreign-exchange market intervention since the end of the Bretton Woods system. Within this period, the "Louvre Accord" intervention policy following the summit meeting in February 1987 stands out as the most ambitious attempt to implement a system of coordinated central-bank intervention by the United States and its trading partners. As such, this policy provides a useful benchmark for considering the effects of intervention policies over other floating-rate periods and, possibly, other markets, as well.

In this paper, I investigate the relationship between occasional interventions and

the behavior of rates, focusing upon the Louvre period. I first show theoretically how this type of intervention policy affects the behavior of the exchange rate. A unique feature of this model is that the behavior of the exchange rate depends directly upon the probability of intervention. I estimate this probability of intervention and the model of exchange-rate behavior. The basic theoretical predictions hold for the DM/\$ exchange rate and, less strongly, for the yen/\$ exchange rate.

The implications of the model are quite intuitive. The Louvre intervention policy set targeted levels for the DM/\$ and yen/\$ exchange rates. As exchange rates deviated from these levels, the Group of Three (G-3) central banks were supposed to intervene to push rates back toward their targeted levels.¹ Since traders were aware of the central bankers' intentions, they expected movements in the exchange rate away from targeted levels to be offset with increasing likelihood as rates drifted from the target. Thus, one implication of the model is that the intensity of intervention induces expected reversion to target levels in the

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¹The Group of Three (G-3) are the United States, Germany, and Japan. Although other industrialized countries at the summit also agreed upon the Louvre Accord, most of the intervention was carried out by these three countries.

determinants of rates and, therefore, the rates themselves. While this type of mean-reversion has previously only been posited, this model shows how the reversion depends directly upon parameters in the intervention policy.² The empirical evidence indeed finds that the conditional mean of the exchange rate depends upon the probability of intervention.

A second feature of the intervention behavior comes from its stabilizing role. Although the central banks intervened with greater intensity as exchange rates deviated from their targeted levels, the interventions appeared to stabilize rates around current levels. As a result, the model predicts that the conditional variance of the fundamental variables, and hence of the exchange rate itself, declines as the exchange rate deviates from its targeted level. This conditional-variance behavior depends upon the intervention probability, a relationship that I also find empirically below.

An alternative description of intervention policy over this period is that central bankers maintained the exchange rate within a given band around the targeted levels. Paul Krugman (1991) has shown that if central banks intervene with certainty at given exchange-rate bands and if the market recognizes this policy, expectations of these interventions will induce nonlinearities in the relationship between the exchange rate and its fundamental determinants. As shown by Lars Svensson (1991), this target-zone model also implies that the conditional variance of the exchange rate will decline as the exchange rate nears the bands. Thus, a finding that the conditional variance falls as the exchange deviates from its target as implied by the intervention model is also consistent with the Krugman target-zone model.

To consider whether target bands are important over this period, I conduct tests of nonlinearities in the exchange rate and fundamentals relationship. Based upon these

tests, I do not find any evidence against linearity despite using quite different measures of fundamental variables. To check whether the lack of evidence for nonlinearities is due to low power of the tests, I estimate the model parametrically and then use this model as a data-generating process to conduct Monte Carlo experiments of the test statistics. These experiments show that nonlinearities induced by targeted bands should have been picked up easily by the tests. In other words, the tests are quite powerful against this alternative. Thus, target bands do not appear to explain the results.

In the absence of target bands, however, the intervention policy itself implies that the exchange rate remains a nonlinear function of its fundamental determinants, raising the question of why these nonlinearities are not detected. Therefore, I conduct another set of Monte Carlo experiments based on the parametric model, but without imposing target bands. The experiments show that the relationship implied by the intervention model is sufficiently linear that the test statistics are likely to be unable to detect them.

Section I below presents the basic theoretical framework. Section II provides the empirical evidence. Section III shows how the occasional intervention model differs from other models such as the Krugman model. Concluding remarks follow.

I. Interventions and the Behavior of Rates

To provide a framework for the investigation, I will begin with a standard asset pricing relationship. Specifically, the asset price depends both upon a set of fundamental variables that influence its contemporaneous demand and supply and upon the expected future asset price. This relationship is given by

$$(1) \quad x(t) = f(t) + \alpha E_t\{dx(t)\} / dt$$

where x is the logarithm of the asset price, f is a composite variable of the determinants of the price, α parameterizes the sensitivity of the asset price to its own expected future change, dx is the change in

²Kenneth Froot and Maurice Obstfeld (1991) and Francisco Delgado and Bernard Dumas (1992) assume that fundamentals follow a mean-reverting Ornstein-Uhlenbeck (OU) process.

the price over the interval of time, and dt is the interval of time. For this equation to explain the exchange rate, x is defined as the logarithm of the foreign-currency price of a unit of domestic currency and f is a measure of its fundamental determinants.³

Since equation (1) represents a first-order differential equation in x , the exchange rate can be solved in terms of fundamentals given the process followed by fundamentals. A standard assumption is that these fundamentals evolve according to a random walk, possibly with drift. I will use this assumption both to show how intervention will alter the behavior of the standard fundamentals process and to contrast the intervention model with conventional ones. This process is

$$(2) \quad d\tilde{f} = \mu dt + \sigma dz$$

where $d\tilde{f} \equiv \tilde{f}(t) - \tilde{f}(t - dt)$, μ is a constant drift term, and $dz = z(t) - z(t - dt)$, where $z(t)$ is a random variable with increments over the interval dt that are independent and normally distributed with zero mean and unit variance. To understand how intervention during the late 1980's would affect a fundamentals process such as (2), I will first describe this policy and then return to the issue of solving the model.

A. G-3 Intervention Policy and Exchange Rates: The Evidence

In 1985, the United States resumed foreign-exchange intervention after a hiatus of five years covering the first Reagan administration. Figure 1 depicts the DM/dollar and the yen/dollar exchange rates for the period 1985-1987. To investigate the relationship between the exchange rates and intervention over this period, these exchange-rate series were combined with series identifying days when foreign-exchange traders observed one of the G-3 central

banks intervening. Intervention accounts were also divided into dollar sales intended to weaken the dollar and dollar purchases to support the dollar. The intervention series were compiled for this study from daily newspaper accounts from *The New York Times*, *Wall Street Journal*, and the *London Financial Times*.⁴ The daily exchange-rate data are reported by the Bank of England as collected by the International Monetary Fund. These rates are quoted in London at 7:00 A.M. EST and are therefore observed before the opening of the U.S. markets.

The figure illustrates the three periods of intervention policy described by Kathryn Dominguez (1990). The first period began following a meeting of the G-5 countries at the "Plaza Meeting" in September 1985 where the governments announced that a fall in the value of the dollar was desirable. Subsequently, the dollar declined dramatically against both the deutsche mark and the Japanese yen. The second period began with the Tokyo meeting on May 5, 1986, in which Japanese officials were concerned that the yen might strengthen too much. By February 1987, the beginning of the third intervention period, official concerns about the weakness of the dollar led to the Louvre Accord, an agreement among central banks to stabilize exchange rates. Yoichi Funabashi (1989) reports the target levels immediately following the Louvre as DM 1.825/dollar and as ¥153.5/dollar. The yen/dollar rate was later rebased to ¥146/dollar. During the months following the Louvre accord, exchange rates appeared to be quite stable relative to the previous two years. The upper right-hand panel of Figure 1 shows this period in more detail, including the midpoint of the yen/dollar targets of ¥149.8/dollar.

Table 1 describes summary statistics of interventions by the Federal Reserve, the

³In standard exchange-rate models, $f(t)$ is the combination of factors that determine the flow supply relative to demand for foreign exchange. In monetary models, such as Michael Mussa (1982), α is the semi-elasticity of money demand.

⁴The exchange rate is determined by the private market demand based upon currently available information. Since the market does not perfectly observe the magnitudes of intervention, these data and not actual intervention data are appropriate for the study. See also footnote 6.

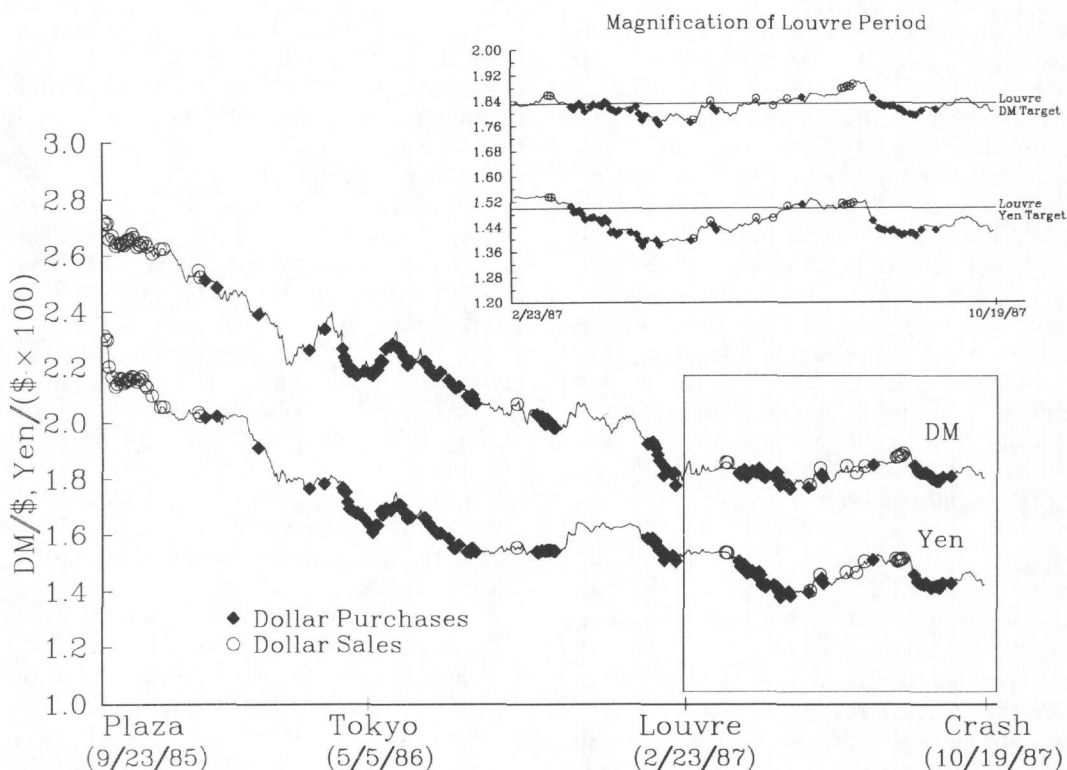


FIGURE 1. DOLLAR EXCHANGE RATES AND G-3 INTERVENTION

TABLE 1—G-3 INTERVENTION, SUMMARY STATISTICS

| Period and bank | Proportion of total days intervened | Average level | | Average level at intervention | | Average level at dollar sales | | Average level at dollar purchases | |
|--|-------------------------------------|---------------|-------|-------------------------------|-------|-------------------------------|-------|-----------------------------------|-------|
| | | DM | Yen | DM | Yen | DM | Yen | DM | Yen |
| <i>Total period (532 observations, 9/23/85–12/31/87):</i> | | | | | | | | | |
| Federal Reserve | 0.077 | | | 2.02 | 162.5 | 2.36 | 194.1 | 1.79 | 141.5 |
| Bundesbank | 0.152 | | | 2.08 | 166.6 | 2.43 | 197.2 | 1.86 | 146.8 |
| Bank of Japan | 0.217 | | | 2.03 | 159.6 | 2.66 | 216.8 | 1.94 | 150.8 |
| Combined | 0.304 | 2.06 | 162.9 | 2.04 | 161.3 | 2.40 | 194.9 | 1.94 | 151.2 |
| <i>Plaza to Tokyo (157 observations, 9/23/85–5/4/86):</i> | | | | | | | | | |
| Federal Reserve | 0.070 | | | 2.63 | 215.4 | 2.67 | 220.1 | 2.17 | 168.5 |
| Bundesbank | 0.178 | | | 2.60 | 211.3 | 2.65 | 215.5 | 2.33 | 184.8 |
| Bank of Japan | 0.172 | | | 2.50 | 201.2 | 2.65 | 216.7 | 2.25 | 175.0 |
| Combined | 0.255 | 2.45 | 195.5 | 2.53 | 203.4 | 2.64 | 215.3 | 2.28 | 178.6 |
| <i>Tokyo to Louvre (205 observations, 5/5/86–2/20/87):</i> | | | | | | | | | |
| Federal Reserve | 0.101 | | | 2.03 | 159.8 | — | — | 2.03 | 159.8 |
| Bundesbank | 0.087 | | | 1.99 | 155.9 | 2.05 | 154.0 | 1.98 | 156.0 |
| Bank of Japan | 0.204 | | | 2.11 | 160.8 | — | — | 2.11 | 160.8 |
| Combined | 0.272 | 2.05 | 159.1 | 2.08 | 159.5 | 2.05 | 154.0 | 2.08 | 159.6 |
| <i>Louvre to crash (169 observations, 2/22/87–10/18/87):</i> | | | | | | | | | |
| Federal Reserve | 0.124 | | | 1.83 | 146.6 | 1.87 | 151.4 | 1.81 | 144.3 |
| Bundesbank | 0.148 | | | 1.82 | 144.8 | 1.85 | 148.7 | 1.80 | 142.5 |
| Bank of Japan | 0.219 | | | 1.81 | 143.5 | — | — | 1.81 | 143.5 |
| Combined | 0.314 | 1.82 | 146.0 | 1.82 | 144.9 | 1.85 | 149.4 | 1.81 | 143.6 |

Notes: The spot exchange rates are the DM/\$ and yen/\$ rates from the International Monetary Fund observed in London at 7:00 A.M. Eastern Standard Time. The intervention data are observations by traders of intervention by one of the Group of Three central banks: the U.S. Federal Reserve, the German Bundesbank, or the Bank of Japan. The combined series is a combination of the three central banks. The columns labeled the "average level" give the average level of the DM/\$ and yen/\$ rates over the sample. "Average level at intervention" reports the average exchange rates at which intervention took place. The columns labeled "average level at dollar sales" and "average level at dollar purchases" give the average exchange rates at which central banks sold and bought dollars, respectively.

Bundesbank, and the Bank of Japan over the full period and the three intervention periods. Central banks intervened frequently during the period. For the full sample, central-bank intervention by at least one of the three central banks, given by the "combined" series, occurred 30.4 percent of the total days. Notably, central banks intervened around a much tighter range of the exchange rate following the Louvre agreement than in previous periods. For example, from the Plaza to the Tokyo meetings, the difference between the average DM/dollar rates where the Fed sold and purchased dollars was 0.6 DM (2.67–2.17) while the same difference following the Louvre Accord was 0.06 DM (1.87–1.81). Overall, the joint pattern of intervention and exchange rates suggests that greater exchange-rate stabilization coincided with more active intervention.⁵

B. Characterizing Occasional Intervention Policy

The evidence in Table 1 and Figure 1 suggests at least three basic features of intervention behavior during the Louvre Accord period. First, most interventions appear to be directed toward preventing exchange-rate movements away from a central level. For example, when the dollar was weaker than its targeted level during the Louvre Accord, interventions were usually dollar purchases, and vice versa.

Second, a small proportion of intervention operations were in the opposite direction. For the interventions above or below the target levels during the Louvre Accord period, most but not all of the interventions were dollar sales or dollar purchases, re-

spectively. Thus, when the dollar was weak some interventions were dollar sales, and vice versa. These interventions may have reflected attempts to stabilize fluctuations around the exchange rate's current levels. Third, although the intervention events may be observable by the market, the magnitudes of these interventions are usually not.⁶

The first two features of intervention may be incorporated into a simple rule. Suppose that the authorities wish to target a level of the exchange rate, defined as x_0 , and that the fundamentals level determining this rate is f_0 . For the sake of exposition, suppose the exchange rate is above the target level so that $\tilde{f} > f_0$. Finally, suppose that central bankers watch carefully the movements in determinants of fundamentals to evaluate the effects upon the equilibrium exchange rate. Then, the rule may be described as follows:

- (3) if \tilde{f} moves away from f_0 , buy domestic currency with probability π^u , do nothing with probability $1 - \pi^u$;

if \tilde{f} moves toward f_0 , sell domestic currency with probability π^d , do nothing with probability $1 - \pi^d$

where $\pi^u > \pi^d$. A symmetric argument holds for fundamentals below f_0 .

This intervention policy clearly incorporates the first two empirical features noted above: (i) rates are targeted back toward their levels with probability π^u , and (ii) exchange-rate movements toward the target level are counteracted with a lower probability, π^d . Below, I will call π^u and π^d the outward and inward intervention probabilities, respectively.

The third feature requires making an identifying assumption about the unobservable magnitudes of interventions. Since central bankers are responding to market forces

⁵The pattern may seem surprising since the interventions' effects upon the domestic money supply are frequently sterilized by the G-3 central banks and since studies such as Lewis (1988) and Kenneth Rogoff (1984) indicate that sterilized interventions should have no effect. However, evidence in Lewis (1995) suggests that the automatic operating procedures by the Federal Reserve may induce a lag in the sterilization process. See Hali Edison (1993) for a survey of intervention studies.

⁶Using a continuous stream of Reuters screen data, Charles Goodhart and Thomas Hesse (1991) find that the intraday volumes of intervention are generally not detectable with the exception of the Bank of Japan interventions for some episodes.

upon the exchange rate, the true intervention magnitudes should reflect the current movement in demand relative to supply for foreign exchange. To characterize this intervention, I will assume that central bankers buy or sell sufficient currency to offset the incipient movement in the exchange rate. To see how this intervention affects the fundamentals behavior, consider the standard random-walk process in (2) typically assumed for fundamentals in the absence of intervention. Suppose that the interval dt equals 1. Then, defining $w(t) \equiv z(t) - z(t - 1)$, the fundamentals process together with intervention can be written as

$$(4) \quad f(t) - f(t - 1) = \tilde{f}(t) - \tilde{f}(t - 1) + a(t) = \mu + \sigma w(t) + a(t)$$

where $a(t)$ is $-[\mu + \sigma w(t)]$ when intervention occurs and 0 otherwise. The magnitude of intervention at time t equals $a(t)$, the quantity of either domestic currency sales (if df is negative) or purchases (if df is positive) that offsets the incipient foreign-exchange demand. The assumptions in (3) and (4) together imply that intervention will target the exchange rate around a given target level (since $\pi^d < \pi^u$) and that the exchange rate will be stabilized around current levels when interventions occur.

C. Intervention-Distorted Fundamentals

To compare the intervention model with other models in the literature that may explain the empirical results found below, the model must be developed in continuous time. For this purpose, it will be convenient to treat the inward and outward probabilities, π^d and π^u , as part of the same continuous probability function. This probability function can be written as $\pi(f)$, where

$$(5) \quad \begin{aligned} 0 < \pi'(f) < \infty & \quad \text{for } f_0 < f < \bar{f} \\ -\infty < \pi'(f) < 0 & \quad \text{for } \underline{f} < f < f_0 \\ \pi(f) < 1 & \quad \text{for all } f \in [f, \bar{f}] \end{aligned}$$

and where π is everywhere continuously differentiable on $f \in [f, \bar{f}]$. The variables f and \bar{f} are the supports of the distribution of fundamentals and can lie anywhere on the real line, including $(-\infty, \infty)$, when $\mu = 0$.

Intervention occurs with probability $\pi(f(t))$. Note that this function has the feature that the outward intervention probability is greater than the inward intervention probability since for $f > f_0$, $\pi'(f) > 0$, and conversely for $f < f_0$. Also, π is clearly minimized at f_0 . Equation (5) also states that the probability of the intervention is strictly less than 1.

The fundamentals process resulting from the distortions introduced by intervention can be derived using the underlying fundamentals process in the absence of intervention in (2) together with the probability function. Appendix A shows that the continuous time limit of the fundamentals process generated by equations (3), (4), and (5) is a diffusion process given by

$$(6) \quad df = \{ \mu[1 - \pi(f)] - \sigma^2 \pi'(f) \} dt + \sigma \sqrt{1 - \pi(f)} dz$$

where dz is the increment to a Wiener process. In Appendix B, I derive the stationary limiting distribution of this process.⁷

This process has an intuitive interpretation. First, the conditional mean is

$$(7) \quad E(df) = \{ \mu[1 - \pi(f)] - \sigma^2 \pi'(f) \} dt$$

and has two components. The first term comes from the effect of the drift term, μ , in the fundamentals process without intervention. If intervention occurs, the change in fundamentals is zero. Therefore, the con-

⁷The Appendix shows that when $\mu = 0$, the asymptotic distribution of (6) with reflecting barriers at (f, \bar{f}) is as follows: $p(f) = A[1 - \pi(f)]$ where $A = \int_{\underline{f}}^{\bar{f}} [1 - \pi(\xi)] d\xi$ is a normalizing constant.

ditionally expected drift is $\mu[1 - \pi(f)]$. The second term appears because interventions prevent incipient exchange-rate movements away from the target level more frequently than movements toward the target. Since decreases in $|f - f_0|$ are less likely to prompt intervention than increases, this policy induces mean-reversion in fundamentals given by $-\sigma^2\pi'(f)$. The gradient $\pi'(f)$ drives the mean reversion because it is locally the difference between the outward and inward probabilities of intervention, $\pi^u - \pi^d$. The other term, σ^2 , measures how much variability in incipient foreign-exchange demand is expected to occur within the period. Thus, in the absence of drift μ , intervention policy will tend to keep the exchange rate from wandering away from the target level.

The conditional variance of the process in (6) is

$$(8) \quad E(df^2) = \sigma^2[1 - \pi(f)] dt.$$

Since the probability of stabilizing intervention increases with deviations from the target level, the conditional variance decreases with these same deviations. The intuition behind the conditional variance is straightforward. In the discrete-time analogue in (4), when intervention occurs, the variance is zero. Therefore, the variance is the probability of no intervention, $1 - \pi$, times the variance in the absence of intervention, σ^2 .

D. Intervention Policy and the Equilibrium Exchange Rate

I can now describe the exchange-rate solution using the intervention-distorted fundamentals process derived above. The exchange-rate solution can be written as a function $x = X(f)$, assumed to be continuous and twice differentiable. In this case, applying Ito's lemma to $X(f)$ using the process of fundamentals in (6) gives

$$(9) \quad dx = (\{\mu[1 - \pi(f)] - \sigma^2\pi'(f)\}X'(f) + \frac{1}{2}\sigma^2[1 - \pi(f)]X''(f)) dt + \sigma\sqrt{1 - \pi(f)}X'(f) dz.$$

Substituting the expected change in the exchange rate in (9) for $E(dx)$ in (1) gives

$$(10) \quad X(f) = f + \alpha\{\mu[1 - \pi(f)] - \sigma^2\pi'(f)\}X'(f) + \frac{1}{2}\alpha\sigma^2[1 - \pi(f)]X''(f).$$

Equation (10) is a second-order differential equation in $X(f)$ and therefore is unique only up to two boundary conditions. These conditions are provided by intervention policy at the boundaries as exchange rates get far away from the target levels. For instance, the assumption that the exchange rate is freely floating except for occasional interventions provides one pair of boundary conditions. Alternatively, the policy discussion during the Louvre period described by Funabashi (1989) also suggests a set of boundary conditions. Specifically, in addition to stabilizing rates around the target levels, the Louvre Accord stated that interventions should keep the exchange rates from exceeding $2\frac{1}{2}$ -5-percent bands around these levels. Krugman (1991) points out that, if intervention is known to keep exchange rates from exceeding a given level, then at this point the expected change in the exchange rate is zero. This result implies that a policy of keeping the exchange rates within the supports of the distribution given by \underline{f} and \bar{f} would imply that

$$X'(\underline{f}) = X'(\bar{f}) = 0.$$

Alternatively, for a policy of free float except for occasional interventions, the supports (\underline{f}, \bar{f}) will be infinite. Below, I will describe the solution with target bands and the effects of these bands becoming arbitrarily large.

Solving the model in (10) requires specifying a probability function for intervention. For now, I will describe the solution for arbitrary probability functions. Given this function and two boundary conditions, the equation can be solved numerically. In Section II, I will estimate this probability function using intervention data.

The upper panel of Figure 2 describes this solution for the case where the bound-

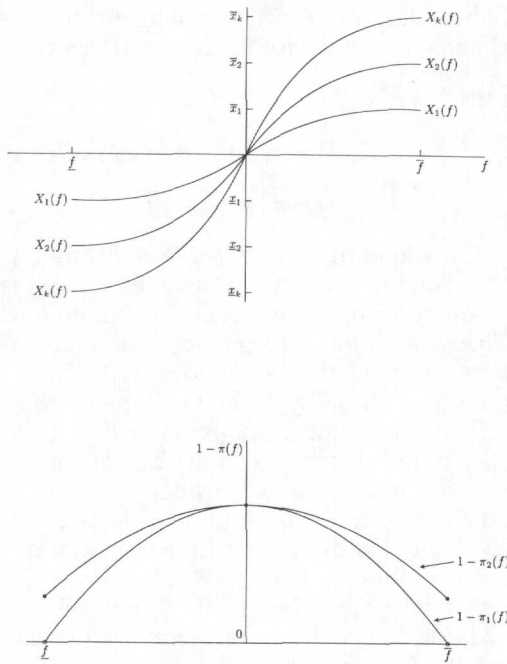


FIGURE 2. EXCHANGE-RATE SOLUTIONS AND THE PROBABILITY OF INTERVENTION

aries of fundamentals are finite, labeled as $X_1(f)$. This solution corresponds to the intervention model when target bands are imposed. For illustrative purposes, the probability function π was assumed to be the uniform distribution, and it was defined over f^2 , to make π symmetric around f_0 , set equal to zero. At the upper boundary, traders know intervention will prevent the exchange rate from depreciating further. Therefore, they bid up the value of domestic currency relative to foreign currency at every lower positive value of fundamentals; and vice versa for negative fundamentals.

This trading behavior comes from the knowledge that intervention will prevent the exchange rate from exceeding the bands, giving the solution in Figure 2 much of its nonlinear form. The lower panel of Figure 2 shows how the exchange-rate behavior depends upon the probability of no intervention, $1 - \pi$. The curve labeled $1 - \pi_1(f)$ corresponds to the exchange-rate solution $X_1(f)$. This probability function describes a

natural case in which the points of reflecting barriers on fundamentals, (f, \bar{f}) , coincide with the level of fundamentals where the probability of intervention, π , is arbitrarily close to 1.

By contrast, the curve labeled $1 - \pi_2(f)$ depicts a case for which the points of reflecting barriers on fundamentals have intervention probabilities significantly far from 1. The exchange-rate solution labeled $X_2(f)$ in the top half of Figure 2 illustrates the resulting exchange-rate solution. Since the probability function does not rise as quickly, the authorities intervene with less intensity. As a result, fundamentals are allowed more variation, resulting in a higher discounted present value of expected future fundamentals when $f > 0$, and a lower present value when $f < 0$. Thus, the exchange-rate bands are wider at (\bar{x}_2, \bar{x}_2) than for the case with a higher probability of intervention, (\bar{x}_1, \bar{x}_1) . As the figure summarizes, for given parameters α , μ , and σ from the fundamentals process, a range of solutions exist that depend upon the probability-of-intervention function.

Equation (10) was also solved for increasingly wider bands of (f, \bar{f}) . As these bands get wider, the exchange-rate function becomes highly linear. Whether bands or a relatively free float are the appropriate boundary conditions is an empirical question that will be investigated below.

II. Empirical Evidence on Intervention Policy and Targeting Rates

The previous section showed that an intervention process characteristic of foreign-exchange intervention policy by the G-3 during the late 1980's would distort the behavior of foreign-exchange supply relative to demand. It also demonstrated how the exchange-rate solution depends directly upon the probability of intervention. This probability can be estimated empirically.

Estimating the probability function in terms of fundamentals, $\pi(f)$, would require knowledge of all of the ingredients that affect demand and supply of foreign exchange, as well as the function that links them together. Since it is well known that

this composite fundamental variable is quite difficult to measure, I will treat it implicitly through the exchange-rate solution. For this purpose, recall that the equilibrium exchange rate is given by the monotonically increasing function, $X(f)$. Thus, in equilibrium, any given fundamentals level, f^* , implies a corresponding exchange rate, and vice versa so that

$$(11) \quad x^* = X(f^*) \Rightarrow f^* = X^{-1}(x^*)$$

where $X^{-1}(x)$ is the inverse function of $X(f)$. Therefore, the probability of intervention may be written as an equilibrium function of the exchange rate by substituting the inverse function in (11) for fundamentals in the probability function:

$$(12) \quad \pi(f^*) = \pi(X^{-1}(x^*)) \equiv \pi^*(x^*).$$

Using the exchange-rate levels where intervention occurred, the probability function can be estimated empirically without specifying the fundamentals variables.⁸ The following subsection provides empirical estimates of the intervention probability as a function of the exchange rate.

A. Estimating the Intervention Probability as a Function of the Exchange Rate

I consider three possible events, defined as I_t , which may occur on any given day, t : $I_t = 0$ for "no intervention," $I_t = 1$ for "intervention to weaken the dollar" (dollar sales), and $I_t = -1$ for "intervention to support the dollar" (dollar purchases). Summary results concerning these series during the period 1985-1987 are provided in Table 1.

To match these observations with the model above, a form for the intervention probability must be specified. This intervention probability appears to be better charac-

terized empirically in units of the level of the exchange rate, $s_t \equiv \exp(x_t)$, rather than the logarithm of the exchange rate, x_t . As in equation (12), this probability can be related back to fundamentals:

$$(12') \quad \pi(f^*) = \pi(X^{-1}(\ln(s^*))) \equiv \pi^s(s^*).$$

Since $\ln(s_t)$ is a monotonic function, estimates of $\pi^s(s_t)$ can be easily mapped into $\pi^*(x_t)$, a procedure I follow throughout the analysis below.

The multinomial logistic distribution over the three possible intervention events provides a relatively good fit of the intervention data. This probability distribution is given by

$$(13a) \quad \ln\{\Pr(I_t = -1)/\Pr(I_t = 1)\} \\ = c_0 + c_1 s_{t-1}$$

$$(13b) \quad \ln\{\Pr(I_t = 0)/\Pr(I_t = 1)\} \\ = g_0 + g_1 s_{t-1}$$

where c_0 , c_1 , g_0 , and g_1 are parameters to be estimated.⁹ The probabilities are based upon the lagged exchange rate in order to minimize potential endogeneity problems. Note that these equations are intended to provide estimates for the model above and should not be viewed as reaction functions. Clearly, additional variables could affect the likelihood of intervention, but these variables should also be present in the exchange rate through the equilibrium relationship in (11).

The parameters in the probability function (13) can be used to determine whether the probability of intervention is increasing with deviations from a target level, as implied by the intervention model. In (13a), a fall in the price of dollars should increase the probability of interventions to buy dollars relative to the probability of interven-

⁸I will also empirically examine the exchange-rate solution allowing for different assumptions about fundamentals.

⁹For details on the multinomial logistic model, see G. S. Maddala (1983).

TABLE 2—PROBABILITY OF INTERVENTION ESTIMATES FOR THE LOUVRE PERIOD

$$\ln\left(\frac{\Pr(I_t = -1)}{\Pr(I_t = 1)}\right) = c_0 + c_1 s_{t-1} \quad \ln\left(\frac{\Pr(I_t = 0)}{\Pr(I_t = 1)}\right) = g_0 + g_1 s_{t-1}$$

A. DM/Dollar:

| Central bank | c_0 | c_1 | g_0 | g_1 | Percentage predicted | $\Pr(I_t = -1)$ | | $\Pr(I_t = 0)$ | | Estimated target |
|----------------------------|--------------|--------------|--------------|--------------|----------------------|-----------------|------------------|----------------|------------------|------------------|
| | | | | | | Mean DM1.82/\$ | Target DM1.82/\$ | Mean DM1.82/\$ | Target DM1.82/\$ | |
| Federal Reserve | 156* (40) | -84* (21) | 127* (34) | -67* (18) | 85.8 | 0.12 | 0.12 | 0.87 | 0.87 | 1.85 |
| Bundesbank | 144* (33) | -79* (18) | 79* (24) | -41* (13) | 85.2 | 0.02 | 0.02 | 0.97 | 0.97 | 1.82 |
| Bank of Japan ^a | — | — | 51* (15) | -29* (8) | 77.5 | 0.14 | 0.14 | 0.86 | 0.86 | — |
| Combined | 125* (27) | -68* (15) | 77* (22) | -40* (12) | 68.0 | 0.05 | 0.05 | 0.94 | 0.94 | 1.83 |

B. Yen/Dollar:

| Central bank | c_0 | c_1 | g_0 | g_1 | Percentage predicted | $\Pr(I_t = -1)$ | | $\Pr(I_t = 0)$ | | Estimated target |
|----------------------------|-----------------|------------------|-----------------------------|------------------------------|----------------------|-----------------|------------------|----------------|------------------|------------------|
| | | | | | | Mean ¥146.0/\$ | Target ¥149.8/\$ | Mean ¥146.0/\$ | Target ¥149.8/\$ | |
| Federal Reserve | 66.7* (22.8) | -0.45* (0.15) | 55.1* (20.6) | -0.35* (0.14) | 87.6 | 0.05 | 0.03 | 0.94 | 0.91 | 147.0 |
| Bundesbank | 58.5* (17.4) | -0.40* (0.12) | 23.0 [†] (12.3) | -0.14 [†] (0.08) | 85.2 | 0.07 | 0.03 | 0.86 | 0.86 | 147.0 |
| Bank of Japan ^a | — | — | 26.8* (7.6) | -0.19* (0.05) | 79.3 | 0.28 | 0.16 | 0.72 | 0.84 | — |
| Combined | 51.1* (13.2) | -0.34* (0.09) | 26.3* (11.5) | -0.16* (0.08) | 69.2 | 0.17 | 0.09 | 0.78 | 0.83 | 150.4 |

Notes: Each equation was estimated by multinomial logit with 169 observations for the period February 22, 1987, to October 18, 1987. The columns headed by coefficients report the estimates, with the standard errors in parentheses. The column labeled "percentage predicted" reports the goodness-of-fit test described in Maddala (1983) and is the percentage of observations correctly predicted by the model. The " $\Pr(I_t = -1)$ " and " $\Pr(I_t = 0)$ " columns give the estimated probabilities of interventions of dollar purchases and no intervention, respectively, at given exchange rates. These exchange rates are either the means over the period (under "mean") or the official target levels (under "target"). (The target level for Japan is the midpoint between the initial and rebased targets.) These probabilities are calculated by substituting the exchange-rate levels into the multinomial logit equations at the top of the table. The final column, "estimated target," gives the estimate of the exchange rate at which the intervention probability is minimized: $s_0 = [\ln(g_1/c_1 - g_1) - c_0]/c_1$.

^aReported estimates do not include $I = 1$ since there were no dollar sales. Equations were: $\ln(\Pr(I_t = 0)/\Pr(I_t = -1)) = g_0 + g_1 s_{t-1}$.

[†]Statistically significant at the 10-percent marginal significance level.

*Statistically significant at the 5-percent marginal significance level.

tions to sell dollars. Therefore, the model predicts that $c_1 < 0$. In (13b), a fall in the price of dollars should reduce the probability of interventions to sell dollars relative to no intervention, implying $g_1 < 0$. Finally, subtracting (13b) from (13a) gives the probability of buying dollars relative to no intervention. A fall in the price of dollars should increase the probability of buying dollars relative to no intervention, implying that $|c_1| > |g_1|$.

The minimum of this probability function can also be used to determine an implied target level: $s_0 = \exp(x_0)$. Minimizing the probability of intervention from equations (13) gives the exchange-rate level:

$$(14) \quad s_0 = \frac{\ln(g_1/(c_1 - g_1)) - c_0}{c_1}$$

Table 2 reports the parameter estimates of the probability function for the yen/dol-

lar rate and for the DM/dollar rate, during the Louvre period.¹⁰ As the results show, the model provides a fairly good fit. The coefficients for all of the central banks and the combined intervention equations are significantly different than zero and have the anticipated signs and magnitudes. The table also reports the estimates of the intervention probabilities implied by substituting the mean exchange-rate levels and the official target levels, respectively, into equation (13). Finally, the last column shows the target levels implied by substituting the estimated parameters into equation (14). For the deutsche mark, the combined minimum intervention probability levels are near the official target of 1.825 DM/dollar and the yen estimate lies near ¥149.8/dollar, the midpoint between the original and the later rebased targets.

In sum, these results corroborate the view that the probability of intervention was an increasing function of the deviation between exchange rates and their target levels following the Louvre Accord. Below, I will use these estimates to examine the relationship between the exchange rate and its implicit fundamentals.

B. Were There Credible Target Bands? Evidence Based Upon Nonlinearity Tests

When target bands are credibly enforced by intervention, the exchange rate will respond less than proportionally to movements in fundamental variables, as illustrated in Figure 2. This basic relationship is consistent with all models that assume known target bands. Without credible bands, however, the exchange rate depends only upon the daily movements of fundamentals including occasional interventions.

In this subsection, I will test for nonlinearities in the exchange-rate function that arise from target bands. The presence of the intervention probability function itself implies nonlinearities, even in the absence of bands. Therefore, I will also consider

whether the exchange-rate model in the absence of bands can provide sufficiently strong nonlinearities to be detected by these tests.

Testing for Nonlinearities.—William Cleveland and Susan Devlin (1988) propose a test of nonlinearities based upon “locally weighted regression” (LWR) against the null hypothesis of a linear regression.¹¹ The test is a modified *F* test of two alternative models. The linear model is a simple linear regression over all sample points and comprises the null hypothesis. The alternative nonlinear LWR model estimates a linear relationship for each data point weighted by a window of its nearest observations as measured by the units of the variables. In this way, the LWR traces out general nonlinear relationships. If the nonlinear relationship provides a better fit than does the linear one, then the sum of squared errors will be lower for this model, and the *F* test will reject the hypothesis that the models are the same. Below, I call this *F* test the C-D (Cleveland-Devlin) test.

This methodology can be used to examine the importance of targeted bands in the intervention model. To see how, note first that the exchange-rate process evolves according to the general form: $dx = m_x dt + \sigma_x dz$, where m_x is the conditional mean and σ_x is the conditional standard deviation, and where, from (9), assuming $\mu = 0$,¹²

$$(15a) \quad m_x = -\sigma^2 \pi'(f) X'(f) + \frac{1}{2} \sigma^2 [1 - \pi(f)] X''(f)$$

$$(15b) \quad \sigma_x^2 = \sigma^2 [1 - \pi(f)] X'(f).$$

¹¹Richard Meese and Andrew Rose (1990, 1991) and Frank Diebold and James Nason (1990) use this test to examine nonlinearities in the level of the exchange rate over different time periods, finding results consistent with this paper.

¹²When μ was allowed to differ from zero in the tests, the estimates were very poorly behaved, apparently due to multicollinearity. However, the empirical analysis will allow for the potential presence of a constant drift term.

¹⁰Lewis (1990) shows that this specification fits the other periods in Table 1 as well.

To construct tests for nonlinearities due to target bands in the conditional mean, it is important to examine the sources of nonlinearities. The mean in (15a) is nonlinear in fundamentals both because the probability $\pi(f)$ is a nonlinear function and because $X(f)$ is nonlinear. As described above, much of the nonlinearity in $X(f)$ derives from the potential effects of target bands. As the bands become arbitrarily large, $X(f)$ becomes relatively linear. However, some linearity remains even in the absence of target bands because of the form of the intervention probability.

To examine whether nonlinearities arising primarily from bands are important while also allowing for nonlinearities due to the intervention probability, I constructed a pseudo-linear null hypothesis. Specifically, note that as $X(f)$ becomes close to linear, $X'(f)$ becomes relatively constant and $X''(f)$ approaches 0. Retaining nonlinearity in the absence of target bands due to the derivative of the intervention probability, this null hypothesis can be written as

$$(16) \quad (x_t - x_{t-1}) = a_0 + a_1 \pi^{*'}(x_{t-1}) + u_t$$

where $a_0 = 0$ and $a_1 = -\sigma^2 [X'(f)]^2$.

Under the null hypothesis that $X'(f)$ is constant, equation (16) represents a linear regression since the derivative of the probability is treated as data. Equation (16) rewrites the probability process as a function of the exchange rate and uses the fact that in equilibrium $\pi'(f) = \pi^{*'}(x)X'(f)$. Since the derivative of the probability is a nonlinear function, $X'(f)$ obviously cannot be literally constant even in the absence of hard target bands. However, I will show below that the degree of nonlinearities induced by the probability function would not be sufficient to reject linearity even though the nonlinearity implied by target bands would be. Therefore, this test should be viewed as a test for the presence of hard bands around the target level, not as a general test of nonlinearities. As the alternative hypothesis, if target bands induce nonlinearities in the exchange-rate process, then

$X'(f)$ will not be a constant and $X''(f) \neq 0$, so that the exchange rate will be a nonlinear function of π^{*} in (16).

Implementing the nonlinearity test requires a measure for fundamentals. Since these variables are unobservable, I use three different measures and check the robustness of the results. The first measure is based upon the monetary model which maintains that exchange rates are determined by relative monetary policies across countries. As daily data on money supplies are not available, I use interest rates that central banks monitor for monetary policy. The rates are the federal funds rate for the United States, the Lombard rate for Germany, and the call money rate for Japan.¹³

The second measure is motivated by Robert Flood and Rose (1993) who identify "virtual fundamentals" as the fundamentals level implied by the discrete-time form of equation (1): $f_t = x_t - \alpha E_t \Delta x_t$. To obtain a measure of $E_t \Delta x_t$, I regressed Δx on the difference between Eurocurrency interest rates for the dollar, deutsche mark, and yen from the London *Financial Times* and used the fitted values. For α , I considered a range from 0 to 3.

Panel A of Table 3, reports marginal significance levels of the null hypothesis that the conditional mean of the exchange rate is linear in the derivative of the probability of intervention. To calculate the probability of intervention series, I use the form of the probability function defined in equation (13) and impose the estimates of the parameters obtained in Table 2 from the combined intervention. Appendix C explains in detail how the probabilities and their derivatives are constructed from the parameter estimates.

The columns in Table 3A specify the fundamental variable used. The Cleveland-

¹³These data are from the Bank of International Settlements. The tests, as well as parametric estimates discussed in the Appendix, were also conducted using the call money rate in Germany without affecting the main conclusions.

TABLE 3—TESTS FOR NONLINEARITIES FROM INTERVENTION MODEL

| | DM/\$ fundamentals | | | Yen/\$ fundamentals | | |
|------------------------------|--------------------|--------------|--------------|---------------------|--------------|--------------|
| | Monetary | Virtual | | Monetary | Virtual | |
| | | $\alpha = 0$ | $\alpha = 3$ | | $\alpha = 0$ | $\alpha = 3$ |
| Marginal significance level: | 0.601 | 0.329 | 0.602 | 0.225 | 0.216 | 0.383 |
| Number of observations: | (104) | (138) | (61) | (75) | (77) | (61) |

B. Probability of Insignificant Cleveland-Devlin Tests

| | DM/\$ | Yen/\$ |
|--|-------|--------|
| <i>Bands Present</i> | | |
| Proportion of times marginal significance level greater than 5 percent: | 0.000 | 0.000 |
| Proportion of times marginal significance level greater than estimate: | 0.000 | 0.000 |
| <i>Bands Not Present</i> | | |
| Proportion of times marginal significance level greater than 5 percent: | 0.862 | 0.954 |
| Proportion of times marginal significance level greater than 10 percent: | 0.775 | 0.911 |
| Proportion of times marginal significance level greater than estimate: | 0.243 | 0.781 |

Notes: For panel A, the entries are the marginal significance levels of the hypothesis that $\Delta x_t = a_0 + a_1 \pi_t^{*'} + u_t$ is linear given the fundamentals in the column. These levels are the minimums over a grid search of neighborhood windows ranging from 0.4 to 0.9 times the sample size, as described in Cleveland and Devlin (1988). The numbers of parentheses are the numbers of observations in the window of the minimum marginal significance level.

Panel B reports the results of Monte Carlo experiments on monetary fundamentals. The intervention model is estimated parametrically as described in the Appendix. Using these estimates and imposing the assumption of bands at the maximum exchange-rate levels over the sample, exchange rates and fundamentals were drawn for the length of the sample and Cleveland-Devlin statistics (C-D) were calculated. The rows of the table report the proportion of times out of 1,000 replications that the C-D F statistic from the generated model had a marginal significance level greater than 5 percent, greater than 10 percent, and greater than the estimates in panel A for "monetary."

Devlin test requires specifying a proportion q of the sample size as the local window around each observation for estimating the LWR. Following Meese and Rose (1991), I conducted each test repeatedly over a grid search from $q = 0.4$ to $q = 1$ in increments of 0.1. The entries in the table report the *minimum* marginal significance level for the F tests found over this grid search. As such, they provide the strongest evidence possible against linearity. The numbers in parentheses are the numbers of observations in the windows.

As the entries show, there is no evidence against linearity for either currency or for any of the extreme measures of fundamentals. This evidence suggests that the exchange rate is close to a linear function in fundamentals. Apparently, the market did

not believe that central bankers would intervene with certainty at the target bands suggested by the Louvre Accord.¹⁴

There are two potential problems with this evidence, however. First, the test may have poor power for detecting nonlinearities arising from the target bands. Second, the derivative of the probability is clearly a nonlinear function of fundamentals, raising the question of why this source of nonlinearity is not detected by the test. I will describe Monte Carlo evidence on these two questions next.

¹⁴ With different methodology, Michael Klein and Lewis (1993) similarly find that the intervention policy of the G-3 central banks following the Louvre Accord was inconsistent with given target bands.

Can the Test Detect the Nonlinearities Implied by Target Bands?—To ask whether the lack of evidence against linearity in panel A of Table 3 was due to a lack of power, I first estimated parametrically the conditional mean in equation (15). The Appendix provides details about this estimation. I then imposed reflecting barriers on the exchange rate at the maximum levels of exchange rates observed over the period. Using this model as the data-generating process, I repeatedly drew observations for sample sizes corresponding to the data set and calculated the C-D statistic 1,000 times.

The first part of panel B of Table 3 (under “bands present”) reports the proportion of times that the statistic exceeded the 5-percent marginal significance level using monetary fundamentals.¹⁵ Panel B also reports the proportion of times in the Monte Carlo experiments that the statistic exceeded the marginal significance level reported in panel A. As the evidence shows, the marginal significance levels of the test for nonlinearities arising from target bands were always less than 5 percent and therefore clearly less than either 10 percent or the estimates in panel B. This evidence indicates that nonlinearities arising from target bands implied by the Louvre Accord should have been easily detected by the C-D test. For this reason, the evidence in panel B suggests that nonlinearities arising from these bands are very unlikely to be present.

Can the Test Detect the Nonlinearities Implied by the Intervention Policy?—The remaining problem with the nonlinearity tests comes from the probability of intervention. The evidence in Table 3A indicates that the exchange-rate function is relatively linear in fundamentals. Yet at the same time the exchange rate is a linear function of the derivative of the intervention probability, itself a nonlinear function of fundamentals.

¹⁵ Virtual fundamentals are measured as an identity in terms of the exchange rate, so parametric estimation of the exchange rate against this measure is not possible.

These two results are not inconsistent, however, if the degree of nonlinearity implied by the probability of intervention without target bands is sufficiently weak that it cannot be detected by the C-D tests. To consider this possibility, I conducted another set of Monte Carlo experiments. I used the same estimated model as above but did not impose the target-zone barriers. I then generated data from this model for the length of the data sample and calculated the C-D statistic 1,000 times.

The second part of Table 3B reports the proportion of times that the marginal significance level exceeded the 5-percent and 10-percent levels and the proportion of times it exceeded the estimate reported in panel A. Interestingly, the marginal significance level exceeded 10 percent at least 77 percent of the time. Thus, even though nonlinearities were present from the probability function, the C-D statistic would not provide rejections at least 77 percent of the time. Even in the case of the DM/\$ rate for which the marginal significance level was 0.601 in panel A, panel B says that exchange rates with nonlinearities arising solely from the intervention probability would have marginal significance levels exceeding this level 24 percent of the time.

This evidence therefore reconciles the apparent inconsistency posed above. The C-D statistic is sufficiently powerful to detect nonlinearities in the exchange rate arising from target bands, but not to detect those arising from the probability of intervention alone. Estimates of this relationship below will verify that the exchange rate is a relatively linear function of fundamentals when the potential nonlinearities arise from the probability of intervention.

C. *The Intervention Model Without Credible Target Bands*

Strictly speaking, the intervention model implies that $X'(f)$ is nonlinear; but the simulations reported in Table 3 indicate that such nonlinearity is very difficult to detect. I therefore estimate the model assuming that $X'(f)$ is constant. This approach has the advantage that the conditional mean and

TABLE 4—JOINT ESTIMATION OF THE EXCHANGE RATE AND INTERVENTION

| Parameters | DM/\$ | | | Yen/\$ | | |
|-------------------------------|-------------------------------|----------------------|--|--------------------------------|-------------------------------|--|
| | Individual | Joint | <i>p</i> value (H ₀ : β = 1) | Individual | Joint | <i>p</i> value (H ₀ : β = 1) |
| <i>a</i> ₀ | -0.018 (0.041) | -0.227 (0.249) | | -0.134 [†] (0.075) | -0.446 (0.566) | |
| (σ <i>X'</i>) ² | 0.021* (0.011) | 0.003* (< 0.001) | | 0.034 [†] (0.019) | 0.003 ^b (0.001) | |
| ln(σ <i>X'</i>) ² | -1.136 (0.855) | -1.179* (0.026) | | -1.498* (0.460) | -1.140* (0.251) | |
| β | 0.480 [†] (0.262) | 1.595* (0.529) | 0.262 | 0.360 (0.240) | 0.924 (0.675) | 0.910 |
| <i>c</i> ₀ | 125.700* (26.690) | 128.460* (3.250) | | 51.140* (13.190) | 49.230* (6.060) | |
| <i>c</i> ₁ | -67.970* (14.510) | -69.600* (1.740) | | -0.341* (0.090) | -0.327* (0.042) | |
| <i>g</i> ₀ | 76.230* (22.490) | 75.210* (1.150) | | 26.100* (11.560) | 24.650* (3.510) | |
| <i>g</i> ₁ | -40.210* (12.160) | -39.610* (0.620) | | -0.161* (0.080) | -0.151* (0.025) | |

Notes: The table reports the estimates of the parameters given on the left-hand side using one-step maximum likelihood for the system of equations given by (13), (16), and (17). The standard errors are in parentheses. The entries under “individual” give the results based upon estimating the conditional mean (16), variance (17), and probability equations (13) separately. Those under “joint” are based on estimating them jointly. Estimates in the conditional-mean equation are multiplied by 100 to convert them into percentage change. The columns labeled “*p* value (H₀: β = 1)” give the marginal significance levels for the hypothesis that β = 1 in the joint equation. All equations are estimated with the combined intervention series from February 22, 1987, to October 18, 1987.

[†]Statistically significant at the 10-percent marginal significance level.

*Statistically significant at the 5-percent marginal significance level.

variance of the exchange rate can be estimated without requiring that fundamentals be specified.

When *X*(*f*) is linear, the conditional moments in (15) reduce to (16) for the mean and

$$(17) \quad u_t^2 = \exp(b_0) [1 - \pi^*(x_{t-1})]^\beta e_t$$

for the variance, where $b_0 = \log[\sigma^2 X'(f)^2]$ and β = 1 and where *e*_{*t*} is assumed to be lognormally distributed with a mean of unity. The form of the probability function was given in equations (13). The derivation of π*(*x*) from the parameters of the logistic function are detailed in Appendix C.

Table 4 reports the estimates of the intervention model in (13) together with the conditional mean in (16) and variance in (17). The “individual” column entries report the results based upon estimating each equation

individually, while the “joint” column entries are the results of estimating the intervention, mean, and variance equations jointly by maximum likelihood.¹⁶ The joint estimation uses the relationship across equations (16) and (17) that exp(*b*₀) = *a*₁. The numbers in parentheses are the standard errors.

The results point to several interesting features. First, the constant *a*₀ is not significantly different from zero at the 5-percent level and is significant at the 10-percent level only for the individual yen/\$ equation. All of the other parameters are significantly different from zero at the 5-percent level

¹⁶The equations were estimated jointly by one-step maximum likelihood, beginning with the initial consistent estimates from the individual equation estimates in Table 3.

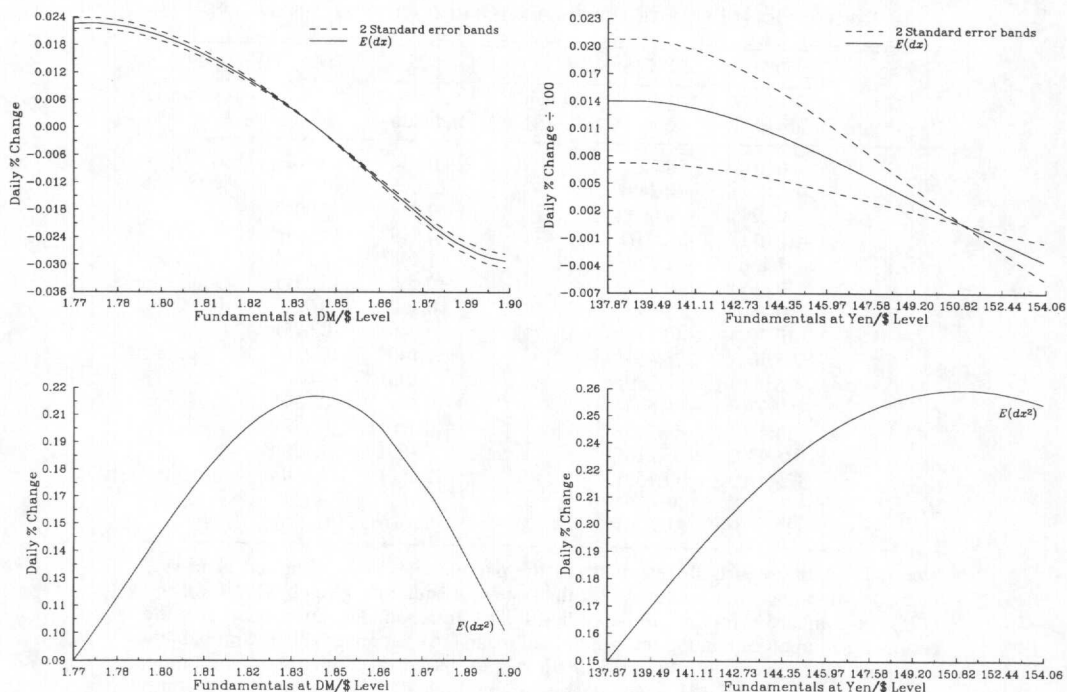


FIGURE 3. CONDITIONAL MEANS AND VARIANCES OF EXCHANGE RATES BASED ON INTERVENTION POLICY

Note: The graphs are constructed from the estimates in Table 4.

with the exception of β for the yen/\$ exchange rate. Furthermore, β is insignificantly different from 1.

Figure 3 illustrates the implications of these estimates for the exchange-rate behavior. The top panels depict the expected change in the exchange rate as a function of the fundamentals level corresponding to different exchange-rate levels. When the DM/\$ rate was 1.77 so that the dollar was weak, the intervention policy meant that the dollar rate was expected to appreciate by 0.024 percent per day. Similarly, when the dollar was at its maximum over the period, intervention induced an expected 0.028-percent depreciation over the next day. To put these estimates into perspective over this period, these changes represented 5 percent and 6.3 percent of the average appreciation and depreciation, respectively, or $5\frac{1}{2}$ times and 7 times the average exchange-rate change. The effects of the intervention on the yen are smaller. The expected apprecia-

tion of the dollar at its low end is only 0.3 percent of the average change, while that of the expected depreciation at the high end is 0.1 percent. These estimates match the description in Yoichi Funabashi (1989) which suggests that interventions against the yen were largely to keep the dollar from falling further.

To give some sense of the relative precision of the estimates, the top panel also plots 2-standard-error bands based upon the conditional-mean equation alone using the joint-equation estimates.¹⁷ These bands suggest that the conditional-mean estimates are more precisely estimated for the deutsche mark.

¹⁷These standard errors were based upon the first equation alone and were calculated as $SD(a_1) \sqrt{(X'X)^{-1}}$ where $SD(a_1)$ is the standard deviation of a_1 and X is the matrix of π' observations. The true standard errors are functions of all the equations in the system and may be larger.

The lower panels depict the estimates of the conditional variances. Recall, however, that the yen/\$ estimates are insignificantly different from zero in Table 4. Interestingly, the DM/\$ estimates show the pattern implied by the model. The variance is minimized at the upper and lower levels of the exchange-rate range. To put these estimates into perspective, the conditional standard deviations of the DM/\$ rate at its lower and upper range, as percentages of the overall standard deviation are 56 percent and 59 percent, respectively. The yen/\$ standard deviation is minimized at the lower range of the exchange rate at 0.39, representing 33 percent of the standard deviation.

III. Relationship Between the Occasional-Intervention Model and Standard Models

I showed above how the exchange rate would be affected by an intervention policy characteristic of the behavior of the G-3 central banks during the late 1980's. The Louvre Accord was followed by a period in which these central bankers were perceived as targeting rates and potentially keeping these rates within bands.

The band behavior is a feature central to all "target-zone" models, first described by Paul Krugman (1991). Furthermore, some of the empirical results found above may seem consistent with these models as well.¹⁸ It is therefore useful to contrast the occasional-intervention model and this popular model.

A. Theoretical Comparison

The Krugman model assumes that fundamentals evolve according to equation (2) but have reflecting barriers at upper and lower levels of fundamentals. Defining $x_K = X_K(f)$ as the solution to equation (1), using equation (2) as the fundamentals process,

and applying Ito's lemma to x_K yields:

$$(18) \quad dx_K = \left[\mu X'_K(f) + \frac{1}{2} \sigma^2 X''_K(f) \right] dt + \sigma X'_K(f) dz.$$

Substituting the expected exchange-rate change from (18) into (1) gives the Krugman solution.

Figure 2 demonstrates how the intervention model differs from the Krugman model when both models incorporate the same target bands as boundary conditions. First, for identical fundamentals boundaries (f, \bar{f}) , the implied bands of the exchange rates with intervention probabilities π_1 or π_2 are (x_1, \bar{x}_1) or (x_2, \bar{x}_2) , respectively. These bands are obviously tighter than the corresponding Krugman bands (x_K, \bar{x}_K) . The intuition for this result is clear. With the occasional intervention policy, the market believes that central bankers will intervene to attenuate movements away from the target level. Therefore, when the exchange rate is above x_o , the present value of the expected future path of excess money supply will be smaller with occasional intervention than without it; and vice versa when the exchange rate is below x_o . Hence, the bands are tighter.

Some of the features of exchange-rate behavior when the fundamentals process is distorted by the occasional intervention resemble those when fundamentals follow a different mean-reverting process, the Ornstein-Uhlenbeck (O-U) process:

$$(19) \quad df = -\rho(f - f_o)dt + \sigma dz.$$

Defining $x_{OU} = X_{OU}(f)$ as the solution generated by (19), Ito's lemma gives the evolution of the exchange rate as

$$(20) \quad dx_{OU} = \left\{ -\rho(f - f_o) X'_{OU}(f) + \frac{1}{2} \sigma^2 X''_{OU}(f) \right\} dt + \sigma X'_{OU}(f) dz.$$

Delgado and Dumas (1992) solve this problem in detail.

¹⁸Svensson (1992) provides a critical survey of the literature.

A comparison of the O-U process in (19) with the intervention-distorted fundamentals process in (6) shows their similarities and differences. The size of the expected change in the exchange rate is minimized at f_0 for both cases. On the other hand, the two processes differ in the effect of intervention upon the variance in the absence of target bands. While the variance of the O-U process is unaffected by intervention, targeting interventions locally to stabilize exchange rates tends to reduce the variance of exchange rates as they move away from fundamentals, as described in (6).

B. Do the Standard Target-Zone Models Generate Nonlinearities Empirically?

In Tables 3 and 4, I showed that the behavior of exchange rates was consistent with an intervention model without target bands. However, these estimates may be picking up relationships captured by one of the two alternative processes above. Therefore, I also tested the null hypothesis of linearities in conditional means based upon these models.

From the Krugman exchange-rate process in (18), the analogue to the intervention-model conditional mean and variance in (15) is

$$(21a) \quad m_{x,K} = \mu X'_K(f) + \frac{1}{2} \sigma^2 X''_K(f)$$

$$(21b) \quad \sigma_{x,K}^2 = \sigma^2 X'_K(f)^2.$$

If bands are not present, then $X'_K(f) = 1$, $X''_K(f) = 0$, and the conditional mean reduces to

$$(22) \quad (x_t - x_{t-1}) = a_0 + u_t$$

where $a_0 = \mu$. Similarly, for the O-U process in (20), the moments are:

$$(23a) \quad m_{x,OU} = -\rho(f - f_0) X'_{OU}(f) + \frac{1}{2} \sigma^2 X''_{OU}(f)$$

$$(23b) \quad \sigma_{x,OU}^2 = \sigma^2 X'_{OU}(f)^2.$$

If $X_{OU}(f)$ is relatively linear as when bands are not present, the conditional mean can be written as

$$(24) \quad (x_t - x_{t-1}) = a_0 + a_1 f_{t-1} + u_t$$

where $a_0 = \rho f_0$ and $a_1 = -\rho X'(f)$. If not, the linearity of equation (24) will be rejected.

Panel A of Table 5 reports the results of C-D tests for nonlinearities arising from these models against the null hypothesis of (22) for the Krugman model and (24) for the O-U model. As with the intervention model, there is no evidence of nonlinearities arising from a target band. All of the marginal significance levels are greater than 10 percent.

To check for the power of the test statistic when bands are present in these models, I conducted similar Monte Carlo experiments as in Table 3. I estimated a parametric version of the models using the monetary fundamentals, as described in Appendix D. The nonlinearity parameters in these estimates are all insignificantly different from zero, corroborating the evidence in panel A of Table 5. I then used these estimates together with the restriction of reflecting barriers on the exchange rate to provide data-generating processes for Monte Carlo experiments on the C-D test statistic. The parametric O-U model for the Japanese yen did not converge, so that these Monte Carlo experiments were not performed.

Panel B of Table 5 reports the results of these experiments. For the available results, the C-D test statistics were never greater than 5 percent, as for the occasional-intervention model. These results indicate that the C-D tests are quite powerful. Therefore, standard target-zone models do not appear to help explain the exchange-rate behavior over the period.

IV. Concluding Remarks and Directions for Future Research

This paper has shown how an intervention policy characteristic of G-3 central banks during the late 1980's would have

TABLE 5—TESTS FOR NONLINEARITIES FROM STANDARD MODELS

| Model | DM/\$ fundamentals | | | Yen/\$ fundamentals | | |
|--------------------|--------------------|----------------|----------------|---------------------|----------------|---------------|
| | Monetary | Virtual | | Monetary | Virtual | |
| | | $\alpha = 0$ | $\alpha = 3$ | | $\alpha = 0$ | $\alpha = 3$ |
| Krugman | 0.608 (60) | 0.118 (134) | 0.123 (138) | 0.279 (75) | 0.640 (134) | 0.140 (61) |
| Ornstein-Uhlenbeck | 0.520 (60) | 0.616 (134) | 0.398 (61) | 0.193 (82) | 0.583 (134) | 0.248 (61) |

B. Probability of Insignificant Cleveland-Devlin Tests When Bands Are Present

| | DM/\$ | Yen/\$ |
|---|-------|--------|
| <i>Krugman model</i> | | |
| Proportion of times marginal significance level greater than 5 percent: | 0.000 | 0.000 |
| Proportion of times marginal significance level greater than estimate: | 0.000 | 0.000 |
| <i>Ornstein-Uhlenbeck model</i> | | |
| Proportion of times marginal significance level greater than 5 percent: | 0.000 | N.A. |
| Proportion of times marginal significance level greater than estimate: | 0.000 | N.A. |

Notes: For panel A, the entries are the marginal significance levels of the hypothesis that $\Delta x_t = a_0 + u_t$ for the Krugman model and $\Delta x_t = a_0 + a_1 f_{t-1} + u_t$ for the Ornstein-Uhlenbeck model are linear given the fundamentals in the column. These levels are the minimum levels over a grid search of neighborhood windows ranging from 0.4 to 0.9 of the sample size, as described in Cleveland and Devlin (1988). The numbers in parentheses are the numbers of observations in the window of the minimum marginal significance level.

Panel B reports the results of Monte Carlo experiments on monetary fundamentals. The models are estimated parametrically as described in Appendix D. Using these estimates and imposing the assumption of bands at the maximum exchange-rate levels over the sample, exchange rates and fundamentals were drawn for the length of the sample, and C-D statistics were calculated. The table reports the proportion of times out of 1,000 replications that the C-D F statistic from the generated model had a marginal significance level greater than 5 percent and greater than the estimates in panel A for "monetary."

altered the behavior of foreign-exchange demand relative to supply and, therefore, exchange-rate behavior. The exchange-rate solution in terms of its intervention-altered fundamental determinants depends directly upon the probability of intervention, which can be estimated from observed intervention behavior. In the paper, this probability was estimated jointly with the exchange-rate behavior.

Interestingly, the evidence finds that intervention policy had a significant effect upon the exchange rate during the Louvre Accord period. On the other hand, the magnitudes of the intervention effects upon the expected change in the exchange rate were relatively small, particularly for the yen/\$ exchange rate. The strongest effect of intervention policy was to reduce the variability

of the DM/\$ exchange rate as it deviated from the target level. Since the Louvre Accord was a period of active intervention policy, these findings suggest that the effects of intervention, even if significant, are likely to be even smaller in other episodes.

APPENDIX A: DERIVATION OF THE INTERVENTION-DISTORTED PROCESS IN EQUATION (6)

Equation (6) in the text provides a continuous-time approximation to the discrete-time process for fundamentals in equation (4) based upon the probability function in equations (3) and (5). To see why, first discretize the state-space of fundamentals and redefine the fundamentals process in (2) as $k (= f)$ and write it as a binomial

process. Thus, let k range over values such that

$$(A1) \quad k_{i+1} - k_i = \xi \quad \forall i$$

where starting at k_i , a small period of time τ later one would have:

$$k_{i-1} \text{ with probability } p$$

$$k_{i+1} \text{ with probability } q$$

where $p = (1/2)[1 - (\mu\tau/\xi)]$ and $q = (1/2)[1 + (\mu\tau/\xi)]$. Define the fundamentals process distorted by fundamentals to be $f(t)$. Then, if at time t , $k_i = f_i$, τ periods later one would have

$$(A2) \quad f_{i-1} \text{ with probability } p[1 - \pi(k_{i-1})]$$

$$f_{i+1} \text{ with probability } q[1 - \pi(k_{i+1})]$$

$$f_i \text{ with probability } p\pi(k_{i-1}) + q\pi(k_{i+1}).$$

Thus, the probability of intervention in the case of movements away from the band is $\pi^u = \pi(k_{i+1})$, while the probability of intervention in case of movements toward the band is $\pi^d = \pi(k_{i-1})$. Clearly, then $\pi^d < \pi^u$ as specified in equation (3).

Next, this discrete-time process may be approximated as a continuous process by taking the limit as the interval of time, τ , becomes small following the steps in Daniel Nelson (1990). For this purpose, consider now the changes in the original fundamentals and the intervention-distorted fundamentals over the interval of time τ , and index these processes according to the time indexes ${}_\tau f_0, {}_\tau f_\tau, {}_\tau f_{2\tau}, \dots, {}_\tau f_{n\tau}$ and ${}_\tau k_0, {}_\tau k_\tau, {}_\tau k_{2\tau}, \dots, {}_\tau k_{n\tau}$. Define the family of sigma algebras generated by the ${}_\tau f_{n\tau}$ sequence as ${}_\tau \Omega_{n\tau}$. Finally, create the continuous-time process as: ${}_\tau f_i = {}_\tau f_{n\tau}$. If it exists, the limit diffusion has the following form:

$$(A3) \quad df = a(f) dt + b(f)^{1/2} dz$$

where

$$a(f) = \lim_{\tau \rightarrow 0} \tau^{-1} E\left\{({}_\tau f_{(n+1)\tau} - {}_\tau f_{n\tau}) \middle| {}_\tau \Omega_{n\tau}\right\}$$

$$b(f) = \lim_{\tau \rightarrow 0} \tau^{-1} E\left\{({}_\tau f_{(n+1)\tau} - {}_\tau f_{n\tau})^2 \middle| {}_\tau \Omega_{n\tau}\right\}$$

(see e.g., S. Karlin and H. M. Taylor, 1981). Defining the standard Weiner process observed over these intervals as ${}_\tau Z_{(n-1)\tau}, {}_\tau Z_{n\tau}, \dots$, the process using equation (4) can be written as follows:

$$(A4) \quad {}_\tau f_{(n+1)\tau} = \begin{cases} {}_\tau f_{n\tau} + \sigma_\tau Z_{(n+1)\tau} + \mu\tau & \text{when no intervention} \\ {}_\tau f_{n\tau} & \text{when intervention.} \end{cases}$$

Note that the conditional mean at each discrete interval is

$$(A5) \quad E\left\{\tau^{-1}({}_\tau f_{(n+1)\tau} - {}_\tau f_{n\tau}) \middle| {}_\tau \Omega_{n\tau}\right\}$$

where ${}_\tau \Omega_{n\tau}$ is the sigma algebra generated by ${}_\tau f_{i\tau}, \forall i$. This process is clearly Markovian. From (A2), the expected evolution of the probability of intervention during τ can also be written as a function of the expected evolution of market fundamentals:

$$(A6) \quad \begin{aligned} \pi({}_\tau k_{(n+1)\tau}) &= \pi({}_\tau k_{n\tau}) + \pi'({}_\tau k_{n\tau})({}_\tau k_{(n+1)\tau} - {}_\tau k_{n\tau}) \\ &\quad + o(\tau) \end{aligned}$$

where $o(\tau)$ collects all terms that approach zero faster than τ . Then, substituting (A6) and (A4) into (A5) and using the fact that, conditional upon no interventions,

$$({}_\tau k_{(n+1)\tau} - {}_\tau k_{n\tau}) = ({}_\tau f_{(n+1)\tau} - {}_\tau f_{n\tau})$$

the conditional mean can be written as

$$(A7) \quad \begin{aligned} E\left\{\tau^{-1}(\sigma_\tau Z_{(n+1)\tau} + \mu\tau) \right. \\ \times [1 - \pi({}_\tau k_{n\tau}) - \pi'({}_\tau k_{n\tau}) \\ \left. \times (\sigma_\tau Z_{(n+1)\tau} + \mu\tau) + o(\tau)] \middle| {}_\tau \Omega_{n\tau}\right\}. \end{aligned}$$

For starting fundamentals level ${}_{\tau}f_{n\tau} = {}_{\tau}k_{n\tau}$, taking the limit as τ goes to zero gives

$$(A8) \quad \lim_{\tau \rightarrow 0} E \left[\tau^{-1} ({}_{\tau}f_{(n+1)\tau} - {}_{\tau}f_{n\tau}) \middle| \Omega_{n\tau} \right] = \mu(1 - \pi(f_t)) - \pi'(f_t) \sigma^2.$$

The conditional variance at each discrete time interval is

$$(A9) \quad E \left[\tau^{-1} ({}_{\tau}f_{k\tau} - {}_{\tau}f_{(k-1)\tau})^2 \middle| \Omega_{k\tau} \right].$$

Substituting (A4) and (A6) into (A9) and taking limits, the conditional variance is given by

$$(A10) \quad \lim_{\tau \rightarrow 0} E \left[\tau^{-1} ({}_{\tau}f_{k\tau} - {}_{\tau}f_{(k-1)\tau})^2 \middle| \Omega_{k\tau} \right] = [1 - \pi(f)] \sigma^2.$$

Next, defining $a(f) = \mu[1 - \pi(f)] - \pi'(f)\sigma^2$, and $b(f) = [1 - \pi(f)]\sigma^2$, and substituting the result into (A3) above yields the form of the diffusion (6). If one restricts $\pi(f)$ to be continuous and differentiable with bounded first derivatives $\forall f \in (f, \bar{f})$, it is straightforward to show that the regularity conditions for nonexplosion given in Daniel Stroock and Srinavasa Varadhan (1979) hold, and (6) is a diffusion process.

APPENDIX B: DERIVATION OF THE DISTRIBUTION OF EQUATION (6)

From Appendix A, write the diffusion process in equation (6) in the following form:

$$df = a(f) dt + [b(f)]^{1/2} dz$$

where

$$a(f) \equiv \mu[1 - \pi(f)] - \sigma^2 \pi'(f)$$

$$b(f) \equiv \sigma^2 [1 - \pi(f)].$$

Then the Fokker-Planck forward equation

provides the transitional density for $f \in (f, \bar{f})$:

$$(B1) \quad \partial p / \partial t = (\partial^2 [b(f)f] / \partial f^2) - (\partial [a(f)p(f)] / \partial f).$$

For a stationary density, $(\partial p / \partial t) = 0$. Therefore, setting the left-hand side of (B1) equal to zero and twice integrating implies

$$(B2) \quad p(f) = m(f) [C_1 M(f) + C_2]$$

where

$$m(f) = b(f) \exp \left[- \int_f^f [2a(\zeta) / b(\zeta)] d\zeta \right]$$

$$M(f) = \int_f^f \exp \left[- \int_f^w [2a(\zeta) / b(\zeta)] d\zeta \right] dw$$

and where C_1 and C_2 are constants of integration that guarantee the following conditions:

(i) $p(f) \geq 0 \quad \forall f \in (f, \bar{f})$

(ii) $\int_f^{\bar{f}} p(\zeta) d\zeta = 1$

(see e.g., Eugene Wong, 1964; Karlin and Taylor, 1981 pp. 219-21). Substituting for $a(f)$ and $b(f)$ yields

$$(B3) \quad p(f) = [(1 - \pi(f))] \exp \{ (2\mu / \sigma^2) f \} \times \left[C_2 + C_1 \int_f^f \frac{\exp [- (2\mu / \sigma^2) \zeta]}{1 - \pi(\zeta)} d\zeta \right].$$

For $\mu = 0$, the conditions above imply $C_1 = 0$ since the integral is not bounded for ζ large. Condition (ii) implies that

$$C_2 = \int_f^{\bar{f}} [1 - \pi(\zeta)] d\zeta$$

giving the distribution

$$p(f) = C_2 [1 - \pi(f)].$$

When $\mu \neq 0$, the distribution is given by solving for C_1 and C_2 in (B3).

APPENDIX C: CALCULATION OF
INTERVENTION PROBABILITY
AND ITS DERIVATIVE

Table 2 reports implied estimates of probabilities of intervention and an estimated target level. The tests in Table 3 use estimates of the probabilities of intervention reported in Table 2. Also, the single-equation and joint-equation estimations in Table 4 use the form of the probability function to construct the likelihood function. In all of these cases, the probability function in (13) was used to provide estimates. This appendix details the form of the probability function, $\pi^*(x)$, and its derivative, $\pi^{*'}(x)$.

For $\pi^*(x)$, first note that in equilibrium, $\pi^*(x) = \pi^*(x(s)) = \pi^s(s)$ as described in equations (12) and (12') in the text. The probability of intervention is $\pi^s(s) = \Pr(I_t = -1) + \Pr(I_t = 1)$. Taking the exponential of equations (13) and rearranging, the probabilities can be rewritten as

$$(C1) \quad \Pr(I_t = -1) \\ = \frac{\exp(c_0 + c_1 s_{t-1})}{1 + \exp(c_0 + c_1 s_{t-1}) + \exp(g_0 + g_1 s_{t-1})}$$

and

$$(C2) \quad \Pr(I_t = 1) \\ = \frac{1}{1 + \exp(c_0 + c_1 s_{t-1}) + \exp(g_0 + g_1 s_{t-1})}.$$

Adding these probabilities together gives the probability of intervention as

$$(C3) \quad \pi^s(s) \\ = \frac{1 + \exp(c_0 + c_1 s_{t-1})}{1 + \exp(c_0 + c_1 s_{t-1}) + \exp(g_0 + g_1 s_{t-1})}.$$

To calculate the derivative of the probability, note that $\pi^{*'}(x) = \pi^{s'}(s(x))s'(x) = \pi^{s'}(s(x))\exp(x)$. Then, $\pi^{s'}$ is calculated by

taking the first derivative of equation (C3):

$$(C4) \quad \pi^{s'} \\ = \frac{(c_1 - g_1)\exp(c_0 + c_1 s_{t-1}) - g_1}{[1 + \exp(c_0 + c_1 s_{t-1}) + \exp(g_0 + g_1 s_{t-1})]^2}.$$

Finally, setting the derivative in (C4) equal to zero, checking second-order conditions, and solving for the s_{t-1} gives the level of the exchange rate with the lowest probability of intervention. The functional form of this exchange rate in terms of the parameters is given in equation (14) in the text, and the estimates are reported in Table 2 under the column "estimated target."

APPENDIX D: PARAMETRIC ESTIMATION

1. *Krugman Model*.—Svensson (1991) shows that the solution to the exchange rate is given by

$$(D1) \quad x = f + \alpha\mu + A_1 \exp(\lambda_1 f) \\ + A_2 \exp(\lambda_2 f)$$

where A_1 and A_2 are constants of integration that depend upon two boundary conditions and the λ_i are roots of the characteristic equation implied by these boundary conditions. The expected change in the exchange rate is

$$(D2) \quad E_t(x_{t+1} - x_t) \\ = [X_K(f) - f]/\alpha \\ = \mu + [A_1 \exp(\lambda_1 f) + A_2 \exp(\lambda_2 f)]/\alpha.$$

For the case when $\mu = 0$ and bands are symmetric, this expression reduces to

$$(D3) \quad E_t(x_{t+1} - x_t) = -\sinh(\lambda f) A$$

where $A = [\alpha \lambda \cosh(\lambda \bar{f})]^{-1}$ and $\lambda = (\sqrt{2}/\alpha)/\sigma$. The conditional variance is

$$(D4) \quad E_t(x_{t+1} - x_t)^2 \\ = \sigma^2 [X'_K(f)]^2 \\ = \sigma^2 [1 - A^* \cosh(\lambda f)]^2$$

where $A^* = 2\alpha\lambda A$. To check for nonlinearities arising from target zones as well as to provide parameters for conducting the Monte Carlo experiments, equations (D3) and (D4) were estimated jointly with the variance of fundamentals assuming innovations to the variances are lognormally distributed. Since α is not identified, the model was estimated for $\alpha = 1$ and $\alpha = 3$ to check for robustness.

For this model, the estimates of the nonlinear term, A , were typically close to zero and never significantly positive. The Monte Carlo experiments used estimates of μ and σ and imposed the condition that f in the boundary condition A corresponds to the strongest value of the exchange rate over the period.

2. *OU Model.*—The solution to the O-U estimates are given in Delgado and Dumas (1991) as

$$\begin{aligned}
 \text{(D5)} \quad X_{OU}(f) &= (f + f_0 \rho \alpha) / (1 + \rho \alpha) \\
 &+ A_1 M[1/2 \rho \alpha; 0.5; \rho(f_0 - f)^2 / \sigma^2] \\
 &+ A_2 M[(1 + \rho \alpha) / 2 \rho \alpha; 1.5; \\
 &\quad \rho(f_0 - f)^2 / \sigma^2] \\
 &\times \sqrt{\rho} (f_0 - f) / \sigma
 \end{aligned}$$

where $M[\cdot; \cdot; \cdot]$ is the confluent hypergeometric function and A_1 and A_2 are again constants of integration. They show that under symmetric bands $A_1 = 0$, and this solution reduces to

$$\begin{aligned}
 \text{(D6)} \quad X_{OU}(f) &= (f + f_0 \rho \alpha) / (1 + \rho \alpha) \\
 &+ A_2 M[(1 + \rho \alpha) / 2 \rho \alpha; 1.5; \\
 &\quad \rho(f_0 - f)^2 / \sigma^2] \\
 &\times \sqrt{\rho} (f_0 - f) / \sigma
 \end{aligned}$$

For the parametric estimation, these nonlinearities were treated as in the case of the

Krugman nonlinearities. In this case, the expected change in the exchange rate is

$$\begin{aligned}
 \text{(D7)} \quad E_t(x_{t+1} - x_t) &= [X_{OU}(f) - f] / \alpha \\
 &= \rho(f_0 - f) / (1 + \rho \alpha) \\
 &\quad + A_{OU} \sqrt{\rho} (f_0 - f) \sinh(\lambda f)
 \end{aligned}$$

where $A_{OU} = 2\lambda A$. This equation was estimated jointly with the variance of fundamentals and the process of fundamentals given by

$$\text{(D8)} \quad f_{t+1} - f_t = -\rho(f_t - f_0) + e_{t+1}.$$

Similarly, the conditional variance is $\sigma^2 [X'_{OU}(f)]^2$. Under the assumption that the nonlinearities of the O-U and the Krugman model are the same, this equation reduces to equation (D4).

As with the Krugman model, the estimates of the nonlinearity parameter, A , were never significantly positive. The Monte Carlo experiments use the estimates of f_{02} , σ , and ρ and impose the constraint that f in A corresponds to the target-zone bands.

3. *Intervention Model.*—Although the intervention model does not have a closed-form solution, its form was approximated for estimation purposes using equation (10) and assuming $\mu = 0$:

$$\begin{aligned}
 \text{(D9)} \quad E_t(x_{t+1} - x_t) &= [X(f) - f] / \alpha \\
 &= -\sigma^2 \pi^{*'}(x) [X'(f)]^2 \\
 &\quad + \frac{1}{2} \alpha \sigma^2 (1 - \pi) X''(f).
 \end{aligned}$$

Treating the target-zone nonlinearities as in the Krugman model, this equation becomes

$$\begin{aligned}
 \text{(D10)} \quad E_t(x_{t+1} - x_t) &= -\sigma^2 \pi^{*'}(x) [1 - A^* \cosh(\lambda f)]^2 \\
 &\quad - \frac{1}{4} \alpha \sigma^2 (1 - \pi) A^* \lambda \sinh(\lambda f).
 \end{aligned}$$

Due to extreme multicollinearity between the first and second terms in (D10), the latter second-order terms were assumed to be zero. The conditional variance is

$$\begin{aligned} \text{(D11)} \quad E_t(x_{t+1} - x_t)^2 \\ &= \sigma^2(1 - \pi)[X'(f)]^2 \\ &= \sigma^2(1 - \pi)[1 - A^* \cosh(\lambda f)]^2. \end{aligned}$$

Equations (D10) and (D11) were estimated jointly with the conditional variance of fundamentals:

$$\text{(D12)} \quad E_t(f_{t+1} - f_t)^2 = \sigma^2(1 - \pi).$$

As with the other models, the estimates of the target-band nonlinearity term, A , were not significantly positive in any of the cases.

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