Journal of International Economics 24 (1988) 109–127. North-Holland

TESTING THE PORTFOLIO BALANCE MODEL: A MULTI-LATERAL APPROACH

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Received February 1986, revised version received April 1987

This paper estimates outside bond demand equations from the portfolio balance model of exchange rate determination for five currencies. The approach used in this paper differs from other structural studies of the portfolio balance model in two fundamental ways. First, while previous studies limit the portfolio choice to domestic assets relative to a composite foreign asset, the model analyzed in this paper decomposes the foreign asset by currency. Second, exploiting the cross-equation correlation that arises from this decomposition provides more efficient estimates of the asset market parameters.

1. Introduction

The portfolio balance model of exchange rate determination has received considerable attention in recent years.[1] According to the portfolio model, the private sector views government issues of debt that are denominated in different currencies as imperfect substitutes. Attention has focused upon this model for primarily two reasons that are related to this view. First, if the portfolio model is correct, conventional thinking about exchange rate determination has been inaccurate in explaining the nominal exchange rate as only the relative price of moneys, i.e. the exchange rate depends upon interest-bearing outside assets as well. Second, the model implies that the authorities could target the exchange rate by swapping the currency denomination of outside debt held by the private sector while leaving the

*I would like to thank two anonymous referees, Jacob Frenkel, John Huizinga, Michael Mussa, and seminar participants at the University of Chicago, the Federal Reserve Board, and New York University for useful comments. I have benefitted from productive discussions with Dale Henderson, Ken Rogoff, Steve Symansky, and Ralph Tryon at early stages in this research. Members of the International Finance Division of the Federal Reserve Board graciously provided the data as well as assistance in using it. I am also grateful to Nadesan Sritharan for research assistance. All errors are mine.

[1]The portfolio balance model of exchange rate determination was developed as the international extension of Tobin's (1969) portfolio balance macroeconomic model. Branson and Henderson (1985) provide a comprehensive survey of the international portfolio balance model literature. Portfolio balance studies that address sterilized intervention policies include Kenen (1982), Girton and Henderson (1977), Henderson (1979, 1984), and Marston (1980).

money supply unchanged, a 'sterilized intervention' in the foreign exchange market.

Overall, bilateral empirical studies of the portfolio balance model have not established a connection between outside bonds and the exchange rate. However, the imprecision in the estimates preclude strong conclusions about the model.[2] In contrast to these bilateral studies, this paper develops and estimates a multi-lateral portfolio balance model. By decomposing demand for foreign assets by domestic residents into a set of bonds denominated in different currencies, the analysis achieves two objectives. First, allowing for substitution among a number of different currencies may uncover new evidence about the portfolio balance model. Second, this multi-lateral approach together with the assumption of rational expectations implies that the model can be estimated more precisely by exploiting cross-equation correlations of forecast errors.

As an additional feature of previous work, most of these studies estimate the portfolio model under the assumption that the error terms are conditionally homoscedastic. But empirical evidence suggests that the ex post realized rates of return may be conditionally heteroscedastic. Therefore, these studies may be reporting incorrect standard errors and estimating the model inefficiently. To investigate this possibility, the empirical results in this paper estimate the portfolio model allowing for conditional heteroscedasticity.

Finally, this paper departs from previous studies by using a different method of aggregating asset demand across domestic and foreign residents. This aggregation is important because data on asset holdings by country of residence are publicly available only for West Germany.[3] With the aggregation, empirical evidence in this paper includes the British pound, the Japanese yen, and the Canadian dollar as well as the German deutschemark. A specification of this aggregation is tested and cannot be rejected in most of the cases.

In the next section, the multi-lateral model and estimation methodology are developed and implemented. Concluding remarks follow.

2. Testing for the portfolio balance channel in a multi-lateral setting

Monetary models of exchange rate determination rely upon the assumption that the private sector views 'outside bonds' the same regardless

[2] Primarily, studies that test for the channel of exchange rate determination implied by the portfolio balance model include Obstfeld (1983), Danker, Haas, Henderson, Symansky and Tryon (1984), and Rogoff (1984). A related literature is the 'outside bonds'-based international CAPM model developed in, for instance, Frankel (1979, 1982). These studies are discussed in section 2.

[3] Danker et al. (1984) use an aggregation for world demand for outside Canadian dollar bonds that requires the real exchange rate to be constant. The aggregation method of this paper requires a constant share of foreign holdings of domestic currency assets.

of currency denomination. By contrast, the portfolio balance model points out that if the private sector considers domestic and foreign currency denominated assets differently, relative supplies of interest-bearing assets, in addition to money supplies, will also determine the exchange rate. Since money demand equations have been investigated extensively in existing literature, this paper will focus upon the role played by the demand for outside bonds denominated in different currencies.

Branson and Henderson (1985) provide a comprehensive survey of the theoretical literature on the portfolio balance model. The essence of the role played by the domestic and foreign demand for outside interest-bearing assets is described briefly here. First, domestic demand for domestic currency-denominated bonds depends upon the domestic nominal interest rate (r), the interest rate on foreign bonds (r^*), the domestic income level (Y) and domestic nominal wealth (W):

$$(B^{\mathrm{d}}/P)=B(r,r^*+D^{\mathrm{e}}(s),Y)(W/P), \tag{1}$$

where

$$B_r>0,\quad B_{(r^*+D^{\mathrm{e}}(s))},\quad B_Y<0,$$

and where P is the domestic price level, s is the logarithm of the domestic exchange rate, D is the forward difference operator, and the superscript 'e' on D indicates the 'expected' change. Also, B_x refers to the partial derivative of B with respect to x. Nominal bond demand rises with the domestic interest rate but falls with the foreign rate of return measured in domestic currency. A rise in income causes an increase in money demand for transactions purposes and, therefore, a decline in bond demand. Eq. (1) incorporates the standard portfolio model assumption that bond demand is homogeneous with respect to nominal wealth.

Similarly, foreign demand for domestic currency assets also depends upon the rates of return and their price, income, and wealth levels:

$$(B^{*\mathrm{d}}/P^*)=B^*(r-D^{\mathrm{e}}(s),r^*,Y^*)(W^*/P^*), \tag{2}$$

where

$$B^*_{(r-D^{\mathrm{e}}(s))}>0,\quad B^*_{r^*},\quad B^*{}_{Y^*}<0,$$

and where an asterisk refers to the foreign component of each variable. Given the demand for domestic currency bonds in these equations, the domestic currency bond market clears when the aggregate of these demand functions equals the outstanding supply, B^{s}. Converting eqs. (1) and (2) into domestic currency units and adding them together gives total demand for nominal domestic currency bonds. Setting bond demand equal to bond supply implies the following equilibrium condition where S is the level of the domestic exchange rate:

$$B = B(r, r^* + D^e(s), Y)W + B^*(r - D^e(s), r^*, Y^*)SW^*. \tag{3}$$

A similar condition holds for foreign currency denominated bonds.

From eq. (3) one can see the essential mechanism for outside bonds to affect the exchange rate. A rise in the supply of domestic currency assets will require a rise in the equilibrium quantity demanded. In the short run, when incomes are fixed, the rise in demand can be achieved with a rise in the domestic interest rate, a fall in the foreign interest rate, or a depreciation of the domestic currency. Under standard stability conditions [see Branson and Henderson (1985), for example] interest rates are primarily determined in money markets so that the exchange rate must rise. From eq. (3), there are two mechanisms for the change in the supply of outside bonds to affect the exchange rate. First, a rise in S will reduce the value of domestic currency assets held by foreigners. Second, given the market's notion of a long-run equilibrium level of the exchange rate, a depreciation of the exchange rate today creates an anticipated appreciation of the exchange rate: $D^e(s)$ falls.

The strength of the effect from either of these mechanisms depends upon how substitutable the private sector views domestic and foreign assets. If highly substitutable, the private sector will essentially be indifferent to a simultaneous rise in domestic currency assets and a fall in foreign currency assets. However, the less substitutable are these assets, the more the exchange rate will have to change in order to equilibrate the asset market. Since a 'sterilized intervention' amounts to exactly such a swap, the portfolio balance model has often been cited as a justification for why intervention can be effective in targeting the exchange rate.

Previous empirical studies of the portfolio balance model such as Obstfeld (1983) and Danker et al. (1984) have estimated bilateral structural models similar to this one. In addition, Rogoff (1984) studies a reduced form of the portfolio balance relationship between Canadian and U.S. dollar assets.[4] These studies approach estimation by investigating basic relationships postulated in the theoretical portfolio balance model while imposing as little restrictions as possible. Another approach, taken in Frankel (1979, 1982) for example, involves placing more structure on the model derived from utility maximization. Although a useful approach for studying such phenomena as exchange rate premia, the approach would require even greater structure to include such variables in the model as the income and the

[4]Also, other studies test reduced-form equations for the exchange rate that would exist under the portfolio model but do not explicitly test for a portfolio balance channel of exchange rate determination. These include Branson, Haltunnen and Masson (1977, 1979) and Dooley and Isard (1979, 1983). Tryon (1983) and Rogoff (1984) survey this literature.

interest rate levels. Therefore, the more agnostic approach was pursued in the present paper.

2.1. The estimation model

In going to the estimation model from the typical model, the foreign currency bond is decomposed into different currency denominations. Focusing upon this decomposition accomplishes two objectives: (a) the multilateral margins of substitution among assets denominated in different currencies can be examined; and (b) the residuals across bond demand equations are correlated so that precision of estimation can be improved by exploiting this correlation.

A bifurcated decision rule employed in the portfolio balance literature provides a convenient means for making the estimation model more tractable.[5] The decision rule is as follows: investors first decide their portfolio allocation between money and interest-bearing assets according to the level of the interest rate; they then choose the currency denomination of interest-bearing assets according to their relative rates of return. In this case, bond demand depends upon the level of the interest rate and the rates of return on assets denominated in different currencies relative to a numeraire currency asset. Under this assumption, the analogue to eq. (1) becomes the following:

$$\left(\frac{B^{ii}}{P^i}\right)=C_0^{ii}\prod_{h=1}^{k-1}\exp\left[C_i^h(r^h-r^*-D^e(s^h))\right]Y_i^{d_1}r_i^{d_2}\left(\frac{W^i}{P^i}\right)^{d_3}\exp(u^{ii}), \tag{4}$$

where B^{ii} is nominal demand by residents of country i for i currency bonds, P^i is the price level in country i, r^* is the nominal interest rate of the numeraire country, r^h is the interest rate of country h, s^h is the logarithm of the hth currency in terms of the numeraire, W^j is the wealth of country j denominated in currency j, Y^i is the income level in country i, and u^{ii} is a disturbance term. As in eq. (1), the level of the interest rate, r, and the income level, Y, enter the bond demand equations inversely to their relationships with money. Hence, d_1 is negative and d_2 is positive. As real wealth rises, the demand for real assets rises so that d_3 is positive. Furthermore, if asset demand is homogeneous with respect to wealth, d_3 should equal one.

The 'own coefficient', C_i^i, is positive since a higher return will cause investors to demand more assets denominated in currency i. The other coefficients differ in sign depending upon whether they are gross substitutes or complements with the assets denominated in the ith currency. As

[5] Branson and Henderson (1985) show how such a decision rule can be derived from utility maximization.

described above, asset demand is homogeneous of degree one in nominal wealth so that $d_3 = 1$.

Imposing a second assumption made in the portfolio balance literature yields a tractable form for the foreign component of asset demand. Domestic residents' demand for money resulting from changes in nominal income or prices is matched by changes in the demand for home currency bonds. As Branson and Henderson (1985) show, the wealth constraint implies that domestic demand for foreign currency assets does not depend upon income in this case.

Since foreign demand for domestic assets does not depend upon the level of the foreign interest rate or income, foreign demand for domestic currency assets can be written simply as a function of rates of return and wealth:

$$S^{ji}B^{ij} = C_0^{ij} \prod_{h=1}^{k-1} \exp\left[C_k^{ij}(r^h - r^* - D^e(s^h))\right](W^j)^{\phi^*} \exp(u^{ij}), \quad i \neq j, \tag{5}$$

where S^{ji} is the jth currency price of the ith currency and u^{ij} is a disturbance term.

Following Danker et al. (1984) and Obstfeld (1983), all foreigners are assumed to have the same demand functions for domestic currency assets. Then total foreign demand for domestic assets can be aggregated over the $k-1$ groups of foreign asset holders:

$$B^{i*} = C_0^{i*} \prod_{h=1}^{k-1} \exp\left[C_h^{i*}(r^h - r^* - D^e(s^h))\right] \sum_{\substack{j=1 \\ j \neq i}}^{k} (W^j S^{ij})^{\phi^*} \exp(u_i^*), \tag{6}$$

where B^{i*} is the aggregate demand by foreigners for bonds denominated in the ith currency. Similarly, the parameters C are common across countries that are foreign to country i and, therefore, contain the superscript asterisk.

Optimally, market demand for domestic currency assets should be estimated for each component of demand in eqs. (4) and (6) with the holdings by domestic and foreign residents as the dependent variable. Unfortunately, data on this breakdown by asset holder are only publicly available for Germany.[6] As a result, demand must be estimated in a form that aggregates across these holders. Total market demand for i-currency bonds comes from aggregating eqs. (4) and (6). Simply adding them together provides an equation that is nonlinear containing two sets of relative rates of returns as right-hand-side variables, a feature likely to introduce strong collinearity.

Conveniently, when the share of domestic currency assets that are held by

[6]For results of empirical disaggregated bond demand equations using this breakdown of German data, see Obstfeld (1983) and Danker et al. (1984). Using confidential data, Danker et al. also estimate domestic and foreign demand equations separately for Japan.

domestic residents is relatively constant over time, the equation may be approximated by a linearization:[7]

$$b^i=\beta_0^i+\sum_{h=1}^{k-1}\beta_h^i[r^h-r-D^e(s^h)]+\gamma_1^i y+\gamma_2^i w+\gamma_3^i w^*+\gamma_4^i r+u^i, \tag{7}$$

where the lower-case variables refer to the logarithms of these variables except for the interest rates. The coefficients on the rates of return and constants include both foreign and domestic parameters. For example, defining q as the share of domestic holdings of total domestic currency assets, $\beta_0\equiv(C_0-C_0^*)q+C_0^*$. The other coefficients, however, depend only upon the function in which they are originally specified. For instance, the parameters on domestic wealth, income and the interest rate depend only upon the domestic demand function.

The coefficient on foreign wealth, $\gamma_3\equiv(1-q)\phi^*$, depends upon both the share of domestic assets held by foreigners and the elasticity of foreign demand for domestic assets with respect to wealth. The foreign wealth coefficient equals one under the homogeneity assumption. But the sign of γ_3 depends upon whether foreigners are net debtors or net creditors in the domestic currency. If they are net debtors, a rise in foreign wealth will prompt them to reduce their indebtedness in domestic currency. So γ_3 is negative and equal to the share of domestic holdings of domestic currency bonds supplied by foreigners, $1-q$.

2.2. The estimation methodology

The unobservability of the forecast of the exchange rate terms, $D^e(s^h)$, hampers estimation of eq. (7). Assuming that expectations are rational, the actual exchange rates can be substituted for the expected terms in eq. (7). Then the error term becomes a composite error that includes a vector of exchange rate forecast errors:

$$b_t^i=R_{t+1}a^i+z_t^i\delta^i+(v_t^i-\varepsilon_{t+1}a^i), \tag{8}$$

[7]For the case of Germany where such data are available, this assumption does not appear unrealistic. Taking the time derivative of the logarithm of the total supply of home currency assets gives the following:

$$[\mathrm{d}(\log(B^s))/\mathrm{d}t]=q_B(t)[\mathrm{d}(\log(B))/\mathrm{d}t]+[1-q_B(t)][\mathrm{d}(\log(B^*))/\mathrm{d}t],$$

where $q_B(t)$ is the share of total holdings by domestic residents. If the shares are assumed constant as a first approximation, integration yields the following relationship:

$$\log(B)=q_B\log(B)+(1-q_B)\log(B^*)+A,$$

where A is a constant of integration that is subsumed into the constant of the estimating equation.

where R_{t+1} is the row vector of relative rates of return, $r_t^k - r_t^* - D^e(s_t^h)$, and a^i is the $(k-1)\times 1$ vector of coefficients on the rates of return. The set of other variables affecting demand for asset i are represented by the row vector, z_t^i, dimensioned by H. In the case of the portfolio model above, these other variables include a constant, domestic and foreign wealth, the domestic interest rate, and the income level so that $H=5$. The coefficients of these other variables are given by the column vector δ^i.

Under the assumption of rational expectations, the error term becomes a combination of the structural error, u_t^i, and the inner product of the $(k-1)\times 1$ forecast error vector, ε_{t+1}, with the rate of returns coefficient vector, a^i. Rational expectations implies that the forecast errors, $\varepsilon^h{}_{+1}$, are white noise. However, the structural error, u_t^i, may in general follow any stationary time series process and, therefore, need not be serially uncorrelated. Since eq. (8) holds for all currency assets, the forecast error vector enters into the composite errors of *all* of the asset equations. Therefore, efficiency can be improved by estimating the equations jointly. This relationship can be seen by stacking the T observations on the $k-1$ asset equations:

$$b=[I\otimes R_{+1}]a+Z\delta+[u-(I\otimes \varepsilon_{+1})a], \tag{9}$$

where b is the stacked vector of assets; R_{+1} and ε_{+1} are the $T\times(k-1)$ matrices of relative rates of return and forecast errors, respectively; I is the $(k-1)\times(k-1)$ identity matrix; Z is the $T(k-1)\times H(k-1)$ matrix of other variables, u is the stacked vector of residuals; and δ and a are the stacked coefficients vectors.

The error terms are correlated across equations since they share common forecast errors. They may also be correlated by the structural disturbances if, for example, policies are coordinated or government deficits are affected by the same world recession. Precise estimation, therefore, should exploit both of these features of the residual. Furthermore, since the right-hand-side variables are likely to be correlated with the residual, consistent estimation of the coefficients requires instrumental variables.

An estimator developed by Cumby, Huizinga and Obstfeld (1983) is particularly well suited for this situation. They show how their estimator, 'two-step two-stage least squares', can efficiently estimate general nonlinear equations with autoregressive and moving average error terms (within a class of GMM estimators). First, the equations are quasi-differenced an appropriate number of times to eliminate the autoregressive component of the error term. Then, Aitken's Theorem is applied to the instrument-transformed equation allowing for the moving average components in the error term.

Returning to eq. (9), this case provides an example in a joint equation setting of the circumstances they describe. To begin with, an assumption about the time series properties of u_t, the structural disturbance, is required.

Some previous studies have assumed that the disturbance follows an AR(1) process.[8] Under this assumption, the procedure calls for quasi-differencing eq. (9) to eliminate the autocorrelation. Next, inspecting the autocorrelation of the resulting residuals indicates that they follow a MA(1) process.[9] Therefore, efficient estimation of this system can be achieved by simultaneously exploiting the cross-equation correlation of the errors together with the time series structure of these errors. Using the joint equation analogue to the Cumby et al. (1983) estimator, 'two-step three-stage least squares' (2S3SLS), meets both of these objectives.

2.3. The data

Estimating the asset demand equations requires data on the 'outside bonds' held by the private sector in the world as a whole. The portfolio balance model focuses upon bonds from outside of the economic sector that arise from government debt. Therefore, the relevant data for estimation should be total debt by all governments in the world broken down by currency denomination. However, such data are not publicly available. Instead, the 'outside bond' data consist of the total government debt broken down by currency for all five countries involved in the study: the United States, the United Kingdom, West Germany, Canada, and Japan. Measuring the bond data in this way implicitly assumes that the governments of other countries such as those of the large number of developing countries make their portfolio decisions similarly to the private sector.

The outside bond data consist of total debt denominated in a particular currency less holdings by government agencies. Holdings by governments include, for example, central bank holdings due to foreign exchange market interventions. The data cover the period from January 1975 through December 1981. Prior to this period, many of the series required for construction of the bond supplies are not available. The assets are measured from the stocks of outstanding official debt, including that of provinces, municipalities, and states for most countries. However, the data series of the United Kingdom does not include local government debt. Since these bond series are stocks of debt measured at different points in time, they will not generally reflect changes in the price of long-term bonds. While it would be preferable to adjust for this problem, the lack of data on the maturity structure for most of the countries makes such a correction difficult. Instead, the analysis follows the theoretical literature in focusing upon the stock of debt and does not address capital gains from bond price changes. The data

[8]See Danker et al. (1984) and Rogoff (1984), for example. Obstfeld (1983) assumes a lagged adjustment bond demand equation that implies a similar process for the structural error term.

[9]Higher order autoregressive and/or moving average processes can be handled in a similar way. See Cumby et al. (1983).

for Canadian, German, and Japanese assets and wealth are from Danker et al. (1984) with some modifications in timing to help maintain the identifying restrictions for estimation. Details concerning the construction of these series are available from the author upon request.

An important consideration in estimating the model is the choice of interest rate series. Ideally, one would want to have an interest rate that accurately reflected the rate of return on the sum total of official debt denominated in a particular currency. The differentials in the rates of return among assets should measure differences that arise solely from currency preferences and not to other factors such as maturity, capital controls, or default risk. For this reason, interest rates on one-month Eurocurrency deposits were chosen as measures of these rates of return.[10]

2.4. Empirical evidence

Eq. (7) was estimated using 2S3SLS, substituting the actual for the expected exchange rates.[11] As discussed above, estimation requires knowledge of the time series process of the structural error, u_t. The bond demand equations were first estimated under the assumption that u_t followed an AR(1) process. In this case, endogenous variables lagged two or more periods are legitimate instruments.[12] However, estimation based upon this assumption provided insignificant parameter estimates for the autocorrelation coefficient. Since incorrectly adjusting for autocorrelation when none exists can potentially remove some of the moving average component from the forecast error, the structural error was assumed white noise.

When the structural disturbance is white noise, endogenous variables that incorporate information lagged one period are legitimate instruments. Hence, the instruments used were a constant, the lagged endogenous variables, income, and the relative rates of return lagged twice.[13] To see why the relative rates of return are lagged twice, note that lagging them one period gives: $r_{t-1}-r^*_{t-1}-s_t-s_{t-1}$. Clearly, then, the relative rates of return lagged one period are not legitimate instruments for estimation since structural disturbances to the bond demand equations are expected to be correlated with the exchange rate under the portfolio model. But for periods $t-2$ and earlier, the relative rates of return are uncorrelated with the current period innovation.

The results of estimating eq. (7) by 2S3SLS are given in table 1. The

[10] The advantage of using these series is that the assets are standardized in terms of all aspects other than currency denomination. The disadvantage is that the short-term rates may not accurately represent the rate of return on longer-term debt.

[11] The computer package is described in Cumby and Huizinga (1984).

[12] Cumby et al. (1983) demonstrate and discuss this point.

[13] In addition, using a number of different instruments including the money supply did not alter the main results. For example, see Lewis (1985).

Table 1

Domestic and foreign aggregate demand for outside assets.
Homoscedastic with respect to instruments, January 1975–December 1981.

$$b=\beta_0+\sum_{h=1}^{k-1}\beta^h(r^h-r^k-D(s^h))+\gamma_1 y+\gamma_2 w+\gamma_3 w^*+\gamma_4 r+u-\sum_{h=1}^{k-1}a_h\varepsilon_{+1,h}.$$

Bond denomination	β_0	Rates of return: Canadian dollar	British pound	German mark	Japanese yen	y	w	w^*	r
Canadian dollar	2.30[a]	−0.08	0.02	−0.03	0.05[b]	−0.05	0.88[a]	−0.08	0.49
	(0.92)	(0.10)	(0.04)	(0.02)	(0.03)	(0.06)	(0.15)	(0.07)	(0.38)
British pound	9.36[a]	−1.14	−0.14	−0.61	−0.01	−0.13	1.76[a]	−1.00	0.65
	(0.85)	(0.96)	(0.35)	(0.56)	(0.21)	(0.99)	(0.75)	(0.61)	(1.43)
German mark	−0.52	0.02	−0.03	0.06	0.04	0.10	1.15[b]	−0.08	−0.01
	(2.48)	(0.12)	(0.05)	(0.06)	(0.06)	(0.09)	(0.52)	(0.11)	(0.44)
Japanese yen	−0.23	−0.14	−0.04	0.02	0.02	0.19	1.01[a]	0.04	0.05
	(0.87)	(0.11)	(0.05)	(0.05)	(0.04)	(0.35)	(0.06)	(0.07)	(0.25)

Method of estimation: two-step three stage least squares.
[a]Indicates significant at the 95 percent level.
[b]Indicates significant at the 90 percent level.

Notes: Mean of left-hand-side variables: C$11.43; BP 10.99; DM 6.07, JY 4.23.
Instruments are lagged values of bond supplies, wealth, and the interest rate; the second lagged values of the premia; and a constant and income.
Rates of return are measured as fractions of 100 percent. Standard errors are in parentheses.

coefficients on the relative rates of return are insignificant for the most part. The only exception is the positive relationship between yen returns and Canadian bonds. Recalling that theoretically the 'own' coefficients should be positive, in only two out of the four cases are these coefficients of the expected sign.

Some other implications of the portfolio model are weakly borne out. In all of the equations except that of DM-denominated bonds, the domestic interest rate coefficients are positive. But they are insignificant in all of the equations. Also, for the Canadian dollar and British pound the income terms are negative as predicted, although insignificant.

The effects of wealth upon asset demand provide stronger evidence for the portfolio model. For all of the asset demand equations, wealth enters with the correct sign. For Canada, Germany, and Japan, the wealth elasticities are significantly different from zero. Furthermore, all of the estimates of the wealth elasticities are relatively close to one. Under homogeneity, the estimated wealth elasticities should equal the share of domestic holdings of domestic currency assets. For all of these countries, the hypothesis that the wealth coefficient is equal to one cannot be rejected.

The sign of the coefficients on w^* depends upon whether the domestic country is a net debtor or a net creditor in assets denominated in domestic currency units. For Germany, the only country where the data on such a breakdown are available, foreigners are net debtors to Germans in DM-denominated assets during this period. Thus, the negative sign on the coefficient of foreign wealth in the German equation is the correct sign.

Also, since $\gamma_2 \equiv q\phi$ and $\gamma_3 \equiv (1-q)\phi^*$, if bond demand functions are homogeneous with respect to wealth, $\gamma_2 + \gamma_3 = 1$. The coefficients correspond roughly to this relationship. For each of the individual equations, the constraint that the wealth coefficients sum to unity was tested with a Wald test. Only the wealth estimates for the Canadian equation rejected this constraint with a marginal significance level of under 10 percent. But the other equations all failed to reject the constraint with marginal significance levels over 80 percent.

The results given in table 1 assume that the error terms are conditionally homoscedastic with respect to the instruments. However, empirical evidence such as in Cumby and Obstfeld (1984) suggests that the residuals may be conditionally heteroscedastic. In this case, these estimates will be inefficient and the reported standard errors incorrect. Therefore, eq. (7) was estimated assuming that the errors are conditionally heteroscedastic. Unfortunately, the covariance matrix was not positive definite for the joint system. Table 2 gives the results from estimating these equations individually. The parameter estimates show relatively little change, while the standard errors are slightly smaller for most estimates. As a result, the coefficient on the relative rate of return for DM assets enters significantly with a negative sign in the

Table 2

Domestic and foreign aggregate demand for outside assets.
Heteroscedastic with respect to instruments, January 1975–December 1981.

$$b=\beta_0+\sum_{h=1}^{k-1}\beta^h(r^h-r^*-D(s^h))+\gamma_1 y+\gamma_2 w+\gamma_3 w^*+\gamma_4 r+u-\sum_{h=1}^{k-1}a_h\varepsilon_{+1,h}.$$

Bond denomination	β_0	Rates of return: Canadian dollar	British pound	German mark	Japanese yen	y	w	w^*	r
Canadian dollar	2.69[a]	−0.11	0.00	−0.03[b]	0.06[a]	−0.04	0.81[a]	−0.04	0.51
	(0.54)	(0.09)	(0.03)	(0.02)	(0.02)	(0.05)	(0.08)	(0.03)	(0.32)
British pound	8.79[a]	−0.57	−0.16	−0.34	−0.00	−0.25	1.26	−0.58	0.87
	(0.75)	(0.85)	(0.38)	(0.5[illegible])	(0.19)	(1.12)	(0.80)	(0.63)	(1.79)
German mark	0.24	0.02	−0.02	0.09	0.02	0.10	1.00[a]	−0.06	−0.02
	(2.10)	(0.12)	(0.05)	(0.06)	(0.05)	(0.93)	(0.44)	(0.10)	(0.53)
Japanese yen	−0.18	−0.16	−0.07	0.01	0.03	0.27	1.00[a]	0.01	0.04
	(0.83)	(0.10)	(0.05)	(0.04)	(0.04)	(0.37)	(0.06)	(0.06)	(0.25)

Method of estimation: two-step two-stage least squares.
[a]Indicates significant at the 95 percent level.
[b]Indicates significant at the 90 percent level.

Notes: Mean of left-hand-side variables: C$11.43; BP 10.99; DM 6.07; JY 4.23.
Instruments are lagged values of bond supplies, wealth, and the interest rate; the second lagged values of the premia; and a constant and income.
Rates of return are measured as fractions of 100 percent. Standard errors are in parentheses.

Canadian dollar bond equation. Overall, however, the results of estimating eq. (7) appear robust to the assumption of conditional homoscedasticity.

An additional consideration in evaluating the results of table 1 are the instruments. Since the parameter results may be sensitive to the instrument set used, a closer inspection of these variables are in order. For the reasons discussed above, the estimation was conducted under the assumption that the structural error was serially uncorrelated. But the constructed asset series are rough, and for some countries some of the components of assets were interpolated from quarterly data. Therefore, it seems reasonable to suspect that serially correlated measurement error might also affect eq. (7). In such a case, even if the structural disturbance were serially uncorrelated the endogenous variables lagged one period would not be legitimate instruments since they are likely to be correlated with measurement errors to asset supplies within that quarter.

To address this possibility, eq. (7) was estimated using endogenous variables lagged three periods as instruments. These results are given in table 3. Allowing for this type of measurement error generally increases the size of the standard errors in the Canadian dollar, German mark, and Japanese yen equations. On the other hand, the precision of estimates in the British pound equation generally improves. The wealth coefficient becomes significantly positive and remains insignificantly different from 1. The standard errors on the parameter estimates for income and the interest rate shrink considerably. Both coefficients are of the theoretically predicted sign. Thus, for the pound sterling case, the previous restriction that the contemporaneous error be uncorrelated with the first lag of the endogenous variables appears to have been too strong.

2.5. Empirical results for a restricted form

The empirical results above indicated little relationship between asset supplies and rates of return. In addition, the domestic variables – the income and the interest rate – were generally insignificant. Therefore, to focus upon the rates of return, an alternative form of the model is next estimated. All countries including the home country are assumed to have the same asset demand functions given by eq. (5). Then, eq. (6) gives the aggregate demand for i-currency assets by omitting the '$j \neq i$' in the summation. Taking the logarithm of this equation gives the equation at the top of table 4 where $\bar{w}$ is the logarithm of world wealth. This equation has the same basic form as eq. (9) where now z_t includes only a constant and world wealth so that $H=2$.

Therefore, table 4 gives the results of again using 2S3SLS to estimate the model. In all of the equations except for Germany the wealth variables are of the correct sign and are significant. However, the hypothesis that the wealth elasticities are equal to one can be rejected in all cases except the Japanese

Table 3

Domestic and foreign aggregate demand for outside assets.
Homoscedastic with respect to the lagged instrument list, January 1975–December 1981.

$$b=\beta_0+\sum_{h=1}^{k-1}\beta^h(r^h-r^*-D(s^h))+\gamma_1 y+\gamma_2 w+\gamma_3 w^*+\gamma_4 r+u-\sum_{h=1}^{k-1}a_h\varepsilon_{+1,h}.$$

Bond denomination	β_0	Rates of return: Canadian dollar	British pound	German mark	Japanese yen	y	w	w^*	r
Canadian dollar	0.50	−0.09	−0.05	0.02	0.01	0.00	1.20[a]	−0.24[a]	−0.19
	(1.34)	(0.05)	(0.03)	(0.03)	(0.02)	(0.05)	(0.24)	(0.12)	(0.46)
British pound	8.08[a]	−0.23	−0.02	−0.10	−0.11	−0.55	0.95[a]	−0.25[a]	−0.06
	(0.18)	(0.17)	(0.14)	(0.10)	(0.11)	(0.27)	(0.16)	(0.11)	(0.55)
German mark	0.26	−0.05	0.01	0.01	0.02	−0.01	1.00[b]	0.06	−0.00
	(2.62)	(0.06)	(0.03)	(0.02)	(0.03)	(0.05)	(0.55)	(0.12)	(0.28)
Japanese yen	−1.09	−0.04	−0.05	0.02	−0.03	−0.25	1.07[a]	0.05	−0.11
	(0.98)	(0.13)	(0.05)	(0.05)	(0.06)	(0.56)	(0.11)	(0.06)	(0.34)

Method of estimation: two-step three-stage least squares.
[a]Indicates significant at the 95 percent level.
[b]Indicates significant at the 90 percent level.

Notes: Mean of left-hand-side variables: C$11.43; BP 10.99; DM 6.07; JY 4.23.
Instruments are bond supplies, wealth, and the interest rate lagged at least three periods; the fourth lagged values of the premia; and a constant and income.
Rates of return are measured as fractions of 100 percent. Standard errors are in parentheses.

Table 4
Portfolio demand for outside bonds: January 1975–December 1981. Homoscedastic with respect to instruments.

$$b^i = c^i + \sum_{k=1}^{k-1} a_k^i (r^k - r^* - D(s^k)) + \Phi w^i + u^i - \sum_{k=1}^{k-1} a_k^i \varepsilon_{+1,k}.$$

		Rates of return (a_k)				
Bond denomination	c_0	Canadian dollar	British pound	German mark	Japanese yen	Φ
Canadian dollar	9.42[a]	−0.70	0.34	−0.86	0.01	0.27[a]
	(0.50)	(0.62)	(0.41)	(0.73)	(0.32)	(0.07)
British pound	8.74[a]	−0.68	0.24	−0.86	0.04	0.37[a]
	(0.44)	(0.82)	(0.47)	(0.70)	(0.42)	(0.07)
German mark	5.75[a]	−0.85	0.17	−0.79	−0.40	0.03
	(0.81)	(0.68)	(0.21)	(0.46)	(0.47)	(0.11)
Japanese yen	−7.10[a]	−1.17	0.53	−1.13	0.35	0.89[a]
	(3.79)	(1.84)	(1.55)	(1.13)	(1.92)	(0.30)

Method of estimation: two-step three-stage least squares.
[a]Significant at the 99 percent level.
Notes: Mean of left-hand-side variables: C$ 11.44; BP 10.99; DM 6.08; JY 4.23.
Instruments: lagged asset supplies and wealth variables, the second lag of the rates of return, and a constant term.
Rates of return are measured as fractions of 100 percent. Standard errors are in parentheses.

yen. Furthermore, the coefficients on the relative rates of return are insignificant in all of the equations. Again, as before, the 'own coefficients' are positive in two out of the four cases.

The insignificance of the coefficients on the individual rates of return leads naturally to the question: Can they jointly help explain the asset demand equations? This hypothesis was posed by testing a joint zero constraint on all of the rate of return coefficients. The chi-squared statistic with sixteen degrees of freedom of 26.8 was rejected at the 95 percent level. Therefore, the relative rates of return do help explain some of the variation in the bond demand equations.

2.6. *Interpretation of the empirical evidence*

Despite attempts to use improved and more efficient empirical techniques, the results of the previous section indicate that estimates of the portfolio balance model remain plagued by imprecision. One possible explanation is that the model is not empirically valid. However, there are also a number of inherent empirical problems that could explain this observation even if the portfolio model were valid.

First, as discussed in the data section, the measurement of bond supplies is necessarily very rough. Despite the careful construction of these series by a number of previous studies, the restricted availability of the data forces one to make a number of assumptions concerning the measurement of the bond supplies. For example, as discussed in the data section, the bond series do not include issues of outside debt by governments or other official agencies outside of the five-country study.

Also, as mentioned in the data section, the bond supplies contain no correction for changes in the price of long-term bonds. Although this issue is not discussed in the theoretical portfolio balance literature, the *value* of bonds should be the empirically relevant measure of bonds. To the extent that the market value of outside bonds differ from the stock of official debt, the bond equations will suffer from an additional source of measurement error.

An additional problem may arise from the level of aggregation. Although the analysis here allows for disaggregation across currency assets, disaggregation in other respects may be important. For example, bond demand may depend upon other domestic variables.[14]

Finally, the measures of the relative rates of return across different currency denominations of assets contain a great deal of noise. Substituting the actual exchange rate for the expected exchange rate, the forecast errors become a part of the residual in estimation. However, empirical evidence from other sources such as Cumby and Obstfeld (1981, 1984) indicate that these series are very noisy. Large forecast errors can therefore contribute to imprecise parameter estimates.[15]

Beyond general interest, the portfolio balance model also provides a motivation for why nonmonetary 'sterilized' foreign exchange market interventions can be used to target the exchange rate. As discussed earlier, how strongly the exchange rate is affected by a swap in the private sector's currency denomination of outside assets depends upon how the private sector views these assets. If very imperfect as substitutes, the swap will require a greater change in the exchange rate. Thus, the small parameter estimates in the bond demand equations in the results above would suggest that sterilized intervention should be highly effective. But for reasons discussed in this section, these parameter estimates are imprecisely estimated, precluding strong statements concerning the effectiveness of intervention policies.

[14]See Danker et al. (1984) for an investigation with alternative domestic assets.

[15]As a check on the signal-to-noise ratio in the relative rates of return, the series were each regressed on the vector of contemporaneous bond supplies. For all but the Canadian dollar return the hypothesis that the coefficients on the asset supplies are jointly zero is rejected at the 5 percent but not at the 10 percent marginal significance level. For the Canadian dollar return, the marginal significance level is 35 percent. The signal-to-noise ratio thus appears to be low.

But the data do provide an opportunity for asking: How much of the change in outside bonds in recent years has come from intervention policy? Estimates of the size of the much-heralded G-5 intervention of 1985 range from 10 to 15 billion. (For example, see *New York Times*, 21 September 1985, p. 7, col. 1.) The intervention was considered to be a relatively large intervention to many market observers. But when compared to the size of outstanding dollar denominated debt from the five countries studied, this figure is relatively insignificant. For December 1982, the size of this outstanding debt was 9.4 trillion dollars. Then, even a relatively large intervention of 15 billion dollars is only about 0.15 percent of the total outstanding supply. Since government deficits have continued to grow since 1982, the fraction of intervention today would be an even smaller number. Hence, from the viewpoint of the portfolio balance model, there has been virtually no intervention policy in recent years since the interventions represent only tiny fractions of the total supply of outside bonds. Rather, most of the change in outside bonds has come from relative growth in government budget deficits across countries.

3. Concluding remarks

This paper has developed and implemented a multi-lateral approach in estimating structural bond demand equations from the portfolio balance model. By exploiting the cross-equation correlation among asset demand functions that arises from assuming rational expectations, the approach gives relatively efficient estimates of the asset market model. The results bear out some general relationships postulated in the portfolio balance model. The strongest significant relationship arises from the effects of wealth upon asset demand. The level of income and the interest rate also generally entered the equations with the correct sign. However, the results were similar to other studies in finding little evidence for the anticipated relationship between bond demand and the measured relative rates of return across currencies. On the other hand, these results appeared to be affected by measurement error in the rates of return.

The results of this paper both shed new light on previous studies of the portfolio model and indicate a direction for future research. First, the estimated standard errors appeared relatively robust to the assumption of conditional homoscedasticity employed in other models. Second, the major evidence for the portfolio model comes from variables other than the rates of return. Finally, by noting that the rates of return contain a low signal-to-noise ratio, the evidence suggests that additional structure must be imposed upon expectations when estimating portfolio balance models.

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