

# Trends in excess returns in currency and bond markets\*

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Recent empirical evidence finds that asset prices are characterized by processes subject to permanent shocks that cumulate into trends. By contrast, theoretical and empirical research usually treats excess returns as following processes subject to purely temporary shocks. The implicit assumption behind this treatment is that the permanent shocks to asset prices cancel out when they are combined to form excess returns. In this paper, we test this hypothesis for foreign exchange and bond returns. Surprisingly, we reject the hypothesis for a wide range of returns, suggesting that there are trends in excess returns. We offer two possible explanations for the presence of these trends. Monte Carlo experiments show that either explanation could produce trends in excess returns in finite samples consistent with our empirical findings.

## 1. Introduction

Many excess returns in financial markets may be written as the return from holding a forward contract on an asset relative to buying the asset in the future spot market. Examples include holding long term bonds and uncovered positions on holding foreign currency denominated bonds. Excess returns can be decomposed into two components: a risk premium and an ex post error in forecasting the risky variable. In most theoretical and empirical applications, researchers have treated the risk premium component as a time varying but mean reverting process with temporary disturbances. Similarly, non-overlapping forecast errors have been treated as transitory white noise processes, an implication of standard rational expectations assumptions.

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Therefore, as the sum of these two terms, excess returns have been assumed to contain only short-lived transitory disturbances.

By contrast, the empirical evidence is mounting that the spot and forward rates are characterized by processes with some permanent disturbances that cumulate into trends. Although in practice researchers may be unable to distinguish between literally permanent disturbances and very slow mean-reverting components, overall the empirical evidence indicates that the degree of persistence in shocks to asset price disturbances is very high.

These treatments imply that the long-run relationship between realized spot rates and their corresponding forward rates must be such that the excess returns they generate do not contain any permanent disturbances. That is, the permanent shocks to spot and forward rates must cancel out when they are combined to form excess returns. Although this restriction is implicit in empirical studies of excess returns, it has yet to be directly tested.

In section 2, we test these restrictions using data from the foreign exchange market and the interest rate term structure for the U.S., the U.K., Germany, and Japan. Surprisingly, we find that the restrictions are rejected for spot and forward exchange rates. We then show that the rejection can be traced back to the behavior of the British pound and the Japanese yen. We are also able to reject the restrictions across currency and bond markets in each country. These results indicate that excess returns in a number of markets contain trends.

Section 3 considers two possible explanations for our findings. Forward rates may be affected by (i) the presence of noise traders in the market that are not rational, or (ii) rational anticipations of infrequent shifts in the process of spot rates. We show that in either case the forecast errors contained in excess returns will appear to follow trends in finite samples. We then use a simple Monte Carlo experiment to show that the presence of such trends is consistent with our empirical findings. Concluding remarks follow.

## 2. Trends in the difference between spot and forward rates

### 2.1. *The basic relationship*

We begin by considering the empirical behavior of excess returns implied by conventional assumptions in the literature. Define  $s_t$  as the logarithm of the spot rate on an asset at time  $t$  and  $f_{t,k}$  as the logarithm of the time  $t$  forward rate on a contract to buy or sell the asset  $k$  periods in the future. Then, the speculative return on a forward contract to buy the asset in the future period is

$$s_{t+k} - f_{t,k} = rp_t + \varepsilon_{t+k}, \quad (1)$$

where  $rp_t$  is the risk premium on this speculative position and  $\varepsilon_{t+k}$  is the

market's error in forecasting the spot rate given information available at time  $t$ .

When the speculative strategy is to hold a contract to sell dollars in the future, then  $s_{t+k}$  and  $f_{t,k}$  are the spot dollar price of foreign currency and its forward rate, respectively. In this case,  $rp_t$  is the 'foreign exchange risk premium' while  $\varepsilon_{t+k}$  is the error in forecasting the exchange rate. When the speculative position is to hold a long term bond and resell it as a shorter maturity bond in the future, then  $s_{t+k}$  is the spot rate yield on the bond at  $t+k$  and  $f_{t,k}$  is the forward rate yield implied by holding the long term bond and selling it at  $t+k$ . In this case,  $rp_t$  is the 'term risk premium' on holding long term relative to shorter term bonds, and  $\varepsilon_{t+k}$  is the error in forecasting shorter term bond returns.

Empirical studies have treated excess returns as processes with temporary disturbances considered to be covariance stationary. In the literature, these processes have been denoted 'I(0)', and we will follow this notation below. Eq. (1) illustrates why we should indeed expect these returns to have only temporary disturbances under conventional assumptions. The excess returns are comprised of two components: a risk premium,  $rp_t$ , and a forecast error,  $\varepsilon_{t+k}$ . Risk premia have been considered stationary on theoretical grounds.<sup>1</sup> And under rational expectations, non-overlapping forecast errors follow white noise, a stationary process.<sup>2</sup> Since the sum of two stationary variables must be stationary, the sum of the risk premium and the forecast error must also be stationary under the assumptions above.

By contrast, the levels of asset prices have been found to contain very persistent shocks, well-approximated as permanent disturbances with unit roots. These processes are covariance stationary after first differencing. Processes with these types of shocks have been denoted 'I(1)' in the literature. Empirical studies have found that the spot and forward rates on the left-hand side of eq. (1) are I(1) variables.<sup>3</sup>

The requirement that both sides of eq. (1) be stationary places restrictions upon the relationship between spot and forward rates. Specifically, if spot and forward rates are I(1), excess returns will only be I(0) stationary when permanent shocks to  $f_{t,k}$  and  $s_t$  cancel out.<sup>4</sup> To see this, we write the spot rate in terms of its permanent and transitory components as

<sup>1</sup>For example, standard models of time-varying risk premia imply that risk premia are stationary since they depend upon the time-series properties of the change in consumption. See Grossman and Shiller (1981) and Backus et al. (1989) for some applications.

<sup>2</sup>Forecast errors for  $k$  periods ahead contain overlapping forecasts and therefore follow a moving average process of order  $k-1$ , also a covariance stationary process.

<sup>3</sup>Meese and Rogoff (1983) and Meese and Singleton (1982) found that exchange rates follow a random walk. Baillie and Bollerslev (1989) test directly for unit roots in exchange rates and find that exchange rates and forward rates are cointegrated. Campbell and Shiller (1987) and Mishkin (1991), among others, find that U.S. interest rates follow I(1) processes.

<sup>4</sup>This is equivalent to the requirement that permanent shocks to  $s_{t+k}$  and  $f_{k,t}$  cancel out because  $s_{t+k} - f_{k,t} = s_t - f_{t,k} + \Delta^k s_{t+k}$  where  $\Delta^k s_{t+k}$  is I(0) stationary.

$$s_t = \sum_{\tau=1}^t \eta_{\tau} + I(0) \text{ terms,} \quad (2)$$

where  $\eta_{\tau}$  is the permanent shock to the spot rate process. We may also write the forward rate in general form as

$$f_{t,k} = b \sum_{\tau=1}^t \eta_{\tau} + \sum_{\tau=1}^t \eta_{\tau}^* + I(0) \text{ terms,} \quad (3)$$

where  $\eta_{\tau}^*$  is another potential permanent shock in addition to the permanent shock to spot rates,  $\eta_{\tau}$ . A *necessary* condition for excess returns to be stationary is that the pair  $\{f_{t,k}, s_t\}$  contains only one trend component,  $\sum_{\tau=1}^t \eta_{\tau}$ ; i.e., that  $\eta_{\tau}^* = 0$  for all  $\tau$ .<sup>5</sup>

To consider this hypothesis, we proceeded in two steps. First, we tested for the number of trends in a vector of spot rates individually. Then, we tested for the number of trends when the vectors of spot and forward rates were combined. If each pair of spot and forward rates share a common trend, the number of trends should not increase when we add forward rates to the system of spot rates. Since it is well-known that the stationary components of spot and forward rates are correlated across countries, we examine systems of several returns in order to increase the power of our tests.

## 2.2. Do foreign exchange excess returns contain additional trends?

We begin by examining forward and spot rates on foreign exchange for various countries. Spot exchange rates, along with one month and three month forward rates, were sampled at the end of the month from Citicorp Database Services for the period 1975 to 1989.<sup>6</sup> The exchange rates studied were the U.S. dollar against the Germany mark, the British pound, and the Japanese yen.

Row 1 of table 1 reports two different tests developed by Johansen (1988) for the hypothesis that there are three or more stochastic trends in the three exchange rates. Both the Trace and Maximal Eigenvalue tests do not reject this hypothesis. Rows 2 and 3 report tests for the number of trends in the three forward rates. Row 2 considers forward rates at the one month horizon and row 3 considers forward rates at the three month horizon. The results in these rows show that we cannot reject the hypothesis that there are three trends in both vectors of forward rates.

<sup>5</sup>A sufficient condition for excess returns to be stationary is that  $\eta_{\tau}^* = 0$  and  $b = 1$ , or equivalently that  $\{s_t, f_{k,t}\}$  are cointegrated with cointegrating vector  $[1, -1]$ . Thus, a failure to reject the restriction we examine need not imply that excess returns are stationary.

<sup>6</sup>These data were provided by Geert Bekaert and Robert Hodrick. For details, see Bekaert and Hodrick (1993).

Table 1  
Johansen tests for number of stochastic trends in exchange rates.<sup>a</sup>

Variables in vector	Number of stochastic trends	Test	
		$T_1^b$	$T_2^c$
1. Spot exchange rates ( $x_t^*$ )	3	13.23	8.81
2. One month forward rates ( $f_t^{x_i}$ )	3	13.13	8.70
3. Three month forward rates ( $f_t^{x_i}$ )	3	12.83	8.41
4. One month forward and spot exchange rates ( $x_t^*, f_t^{**}$ )	6	118.00 <sup>d</sup>	53.13 <sup>d</sup>
	5	64.91	36.83 <sup>d</sup>
	4	28.09	15.20
5. Three month forward and spot exchange rates ( $x_t^*, f_t^{**}$ )	6	115.70 <sup>d</sup>	49.46 <sup>d</sup>
	5	66.23 <sup>d</sup>	37.91 <sup>d</sup>
	4	28.31	15.63

<sup>a</sup>All systems are for \* = £, DM, ¥. Tests are based upon AIC information criterion choice of one lag in VAR system.

<sup>b</sup>Johansen 'Trace test'.

<sup>c</sup>Johansen 'Maximal Eigenvalue test'.

<sup>d</sup>Significant rejection at the 5% confidence level.

Rows 4 and 5 report the test statistics for the hypothesis that the system of spot rates and forward rates contain at least four, five, and six trends, respectively. If the null hypothesis of  $\eta_\tau^* = 0$  for all  $\tau$  holds true, then the spot and forward rates for each currency will share the same trends. In this case, given the results in rows 1 to 3, the number of trends will remain the same at three. However, if forward rates contain additional trend components, then the number may increase. The statistics in row 5 show that we can reject the hypothesis of five independent trend components at the 95% confidence level for systems of spot and three month forward rates. The hypothesis is also rejected with one month forward rates using the Maximum Eigenvalue test. However, we cannot reject the hypothesis of four trends. This result suggests that at least one of the excess returns in the system is non-stationary.

In summary, table 1 provides evidence that forward exchange rates follow trends in addition to those followed by spot rates. Of course, these results are subject to the caveat that the Johansen tests may not be powerful enough to reject the presence of the additional trend.

### 2.3. Are the results robust?

To investigate the power of the Johansen tests and other assumptions about the data which may affect the test statistics, we conducted a number of Monte Carlo experiments. These experiments were constructed to generate spot rate processes with the same variance in their permanent components as we observe in the exchange rate data. From these permanent trend components in spot rates, we generated systems of forward rates with different

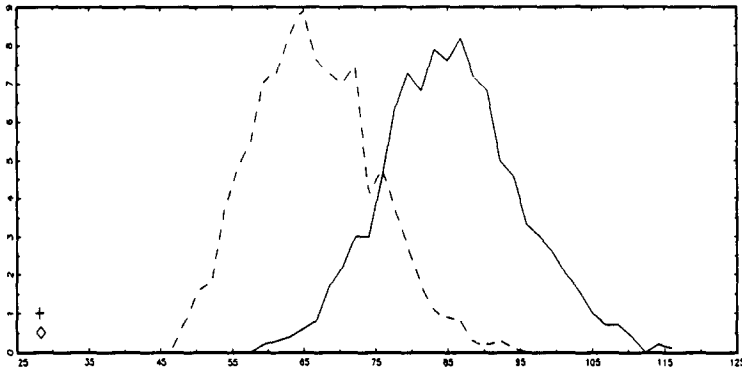


Fig. 1. Monte Carlo distribution of the Trace test.

*Notes:* The statistics test for the presence of 4 trends in the 6-dimensional vector of forward and spot exchange rates when in fact there are 3 in the generated data.

- empirical distribution of test statistics based on a VAR of order 1.
- - - empirical distribution of test statistics based on a VAR of order 3.
- + test statistics for 1-month forward rates reported in table 1.
- ◇ test statistics for 3-month forward rates reported in table 1.

numbers of trends. Therefore, we knew by the construction of each experiment the true number of trends in the artificial forward rates. We then calculated both versions of the Johansen test. Repeating this process 1,000 times, we generated an empirical distribution for the test statistics where the number of trends is known. Details of these experiments are presented in Evans and Lewis (1992).

With the empirical distribution of the Johansen test statistics generated by these Monte Carlo experiments, we can examine the power of the tests to reject additional trends. In other words, we can ask whether these tests would fail to reject the hypothesis of a given number of trends when it should reject. Fig. 1 shows the empirical distribution of Johansen's Trace test for four trends when (by construction) there are only three trends in the data. Because the exchange rate data were used to parameterize this experiment, the Monte Carlo results can be compared to the test statistics in rows 4 and 5 of table 1, where we tested for four trends.

Fig. 1 answers the question: If we test for four trends, but only three are truly present in the system, how likely are we to find the test statistics in table 1? The figure depicts two cases, representing two different assumptions about the order of the VAR used to construct the Johansen tests. Raising the number of lags in the VAR from one to three shifts the empirical distribution to the left but not enough to account for the results reported in table 1. The probability of observing 28.09, the statistic when spot exchange rates are

combined with one-month forward rates when in fact there are only three stochastic trends, is considerably less than the 1% marginal significance level of 51.5 or 53.2 for the two empirical distributions. Therefore, the Johansen test appears to have a good deal of power to reject four trends when only three trends are present. The Maximum Eigenvalue tests also appear to be very powerful. In Evans and Lewis (1992) we show that there is a minuscule probability of finding the statistics reported in table 1 when in fact only three trends were present.

In addition to these experiments, we also conducted experiments that: (1) tested for different numbers of trends holding the true trend number constant; (2) allowed for different numbers of trends; and (3) allowed for heteroskedasticity rather than homoskedasticity. One important result that emerged from these experiments was that ignoring the presence of heteroskedasticity tends to bias the tests statistics upwards. Since exchange rates are known to be heteroskedastic, this result suggests that the statistics reported in table 1 are biased upwards.

In summary, our Monte Carlo results indicate that the test statistics obtained in table 1 based upon the Johansen distribution assuming homoskedasticity are too high. The results in the table are therefore biased toward finding *too few* rather than *too many* trends. We conclude that there is strong evidence of statistically significant trends in forwards rates relative to spot rates.

#### 2.4. Do trends arise in individual currencies?

We now turn to consider whether the trending deviations between spot and forward rates implied by the results in table 1 are associated with a particular exchange rate. We will exploit the fact that covered interest parity holds to combine information contained in interest rates and exchange rates. Let  $R_t^i$  be the interest rate on deposits denominated in currency  $i$ ,  $x_t^i$  the logarithm of the exchange rate of currency  $i$  (expressed as foreign currency price of a dollar), and  $f_t^{x^i}$  the forward rate for future delivery of the exchange rate  $x_t^i$ . The covered interest parity relationship can be written for dollar deposits relative to the domestic currency  $i$  deposits as

$$f_t^{x^i} = R_t^i + x_t^i - R_t^{\$}. \tag{4}$$

Thus, arbitrage ensures that the forward rate is a linear combination of the current spot exchange rate  $x_t^i$ , the current interest rate on dollar bonds,  $R_t^{\$}$ , and the interest rate on domestic currency  $i$  bonds,  $R_t^i$ .

Using this parity condition, we can evaluate whether the deviation between the spot and forward rates implied by the results in table 1 is associated with a particular exchange rate. If spot and forward rates share the same trend,

Table 2

Johansen tests for number of stochastic trends in spot rates and forward rates individually.<sup>a</sup>

Variables in vector	Number of stochastic trends	Assets <i>i</i>					
		A. British pound		B. German mark		C. Japanese yen	
		$T_1^b$	$T_2^c$	$T_1$	$T_2$	$T_1$	$T_2$
One month <sup>d</sup>							
1. Spot rates	3	22.84	11.66	34.78 <sup>e</sup>	28.48 <sup>e</sup>	24.41	21.18
	2	11.18	10.77	6.31	6.30	3.23	2.79
2. Forward rates	3	23.64	11.94	31.67 <sup>e</sup>	25.53 <sup>e</sup>	25.57	21.84 <sup>f</sup>
	2	11.70	11.31	6.14	6.14	3.72	3.25
Three month <sup>a</sup>							
3. Spot rates	3	23.62	11.87	31.65 <sup>e</sup>	25.53 <sup>e</sup>	25.54	21.81 <sup>e</sup>
	2	11.75	11.36	6.12	6.12	3.73	3.26
4. Forward rates	3	22.07	12.24	19.94	14.57	19.76	16.91

<sup>a</sup>Tests are based upon AIC information criterion choice of three lags in VAR system.<sup>b</sup>Johansen 'Trace test'.<sup>c</sup>Johansen 'Maximal Eigenvalue test'.<sup>d</sup>Interest rate spots are for 1 month maturities.<sup>e</sup>Significant at the 90% confidence level.<sup>f</sup>Significant at the 95% confidence level.<sup>a</sup>Interest rate spots are for 3 month maturities.

then the forward premium shares this same trend as well. Since  $f_t^{x^i} - x_t^i = R_t^i - R_t^s$ ,  $R_t^i$  and  $R_t^s$  can have at most one shared independent trend in addition to the trend in the spot exchange rate. Therefore, the greatest number of independent trends that the spot rate, domestic and foreign interest rates can contain under the null hypothesis is two. Based upon this observation, we examined whether additional trends are present by testing for the number of trends in the vector of the domestic interest rate, the exchange rate, and the U.S. interest rate.

Table 2 reports the Johansen test statistics for the three variable systems. The interest rates are Eurocurrency deposits obtained from Harris Bank. We sampled one month spot rates on deposits and the forward rate on a one month deposit for delivery one month in the future.<sup>7</sup> The table reports tests for the number of trends in systems of one and three month interest rates

<sup>7</sup>Using the linearized term structure relationship from Campbell and Shiller (1991) for the case of pure discount bonds as we have here, the forward rate on a  $k$  period bond contracted for trade in  $n$  periods is:  $[(k+n)R_{t,k+n} - nR_{t,n}]/k$ , where  $R_{t,j}$  is the rate on a  $j$  period deposit at time  $t$ . In this paper, we only consider the case where  $k=n$  for one month and three month deposits, so that  $F_{t,k} \equiv [2kR_{t,2k} - kR_{t,k}]/k$  for  $k=1,3$ . Some of these deposits were available for earlier periods than for exchange rates. In the combinations considered below, we used the longest time series of data available.



separately. These tests are conducted for systems of the spot rates in rows 1 and 3 and for the forward rates in rows 2 and 4. As the table shows under column A, we cannot reject the hypothesis that three trends are present in the U.K. at either maturity. For the Japanese yen in Column C, the Trace test for three trends is not rejected at either maturity, although the Maximal Eigenvalue test is rejected for the three month rates. These findings suggest that the pound and yen forward rates contain a trend not found in the corresponding spot exchange rates. Since the results in table 1 found an additional trend in at least one of the forward rates, these results suggest that the additional trend in the pound/dollar returns may be shared with the yen/dollar returns.

### 2.5. *Do trends arise across foreign exchange and bond markets?*

The results in tables 1 and 2 suggest that trends additional to those in spot rates can arise across currencies. They also suggest that bond markets may provide information about these trends. To examine whether the additional trends are detectable in foreign exchange and bond markets, we combined the spot rates on the domestic interest rate, the exchange rate, and the U.S. interest rate  $(R^i, x^i, R^S)$ , with their forward rates  $(f^{R^i}, f^{x^i}, f^{R^S})$  where  $f^{R^i}$  is the forward interest rates in currency  $i$ . Since the relationship between spot and forward rates in (1) applies equally to interest and exchange rates, we may use the same approach to test for the presence of additional trends in  $(f^{R^i}, f^{x^i}, f^{R^S})$  as we did when we considered forward exchange rates alone. Specifically, if there are no additional trends in forward interest rates, we should find the same number of trends in the systems of spot and forward rates as we did for the spot and forward rates separately.

Table 3 reports the results of the Johansen test for the number of trends in  $(R^i, x^i, R^S, f^{R^i}, f^{R^S})$ . We exclude the forward rate exchange rate from this system since it is a linear combination of the spot exchange rate and interest rates by covered interest parity (4). The top panel provides the test statistics for the one month rates while the lower panel provides those for the three month rates. As the statistics show, we can reject the hypothesis of three or more stochastic trends only in the one month returns for the Japanese yen. For the three month returns, we cannot reject the presence of four trends in any of the currencies.

Table 3 supports the findings of additional trends between spot and forward rates found in tables 1 and 2. However, these results show that the additional trends appear across currency and bond markets as well. The combined evidence in tables 1 to 3 indicates that exchange rates and interest rates contain trends additional to those assumed in the existing empirical literature.

Table 3

Johansen tests for number of stochastic trends in spot rates and forward rates jointly.<sup>a</sup>

Variables in vector	Number of stochastic trends	Assets <i>i</i>					
		A. British pound		B. German mark		C. Japanese yen	
		$T_1^b$	$T_2^c$	$T_1$	$T_2$	$T_1$	$T_2$
1. One month <sup>d</sup>	5	130.87 <sup>f</sup>	71.34 <sup>f</sup>	92.01 <sup>f</sup>	48.86	159.74 <sup>f</sup>	92.00 <sup>f</sup>
	4	59.53 <sup>f</sup>	43.44 <sup>f</sup>	43.14	24.80	67.75 <sup>f</sup>	36.40 <sup>f</sup>
	3	16.09	9.69	18.34	12.60	31.34 <sup>f</sup>	27.57 <sup>f</sup>
2. Three month <sup>g</sup>	5	75.02 <sup>f</sup>	54.53 <sup>f</sup>	80.28 <sup>f</sup>	50.59 <sup>f</sup>	54.68	31.83 <sup>e</sup>
	4	20.49	11.57	29.69	17.31	22.86	12.09
	3	8.92	5.23	12.38	9.99	10.77	4.98

<sup>a</sup>Tests are based upon AIC information criterion choice of three lags in VAR system. These systems include  $f^{RJ}$ ,  $f^{R^f}$ ,  $R^j$ ,  $x^j$ ,  $R^S$ .

<sup>b</sup>Johansen 'Trace test'.

<sup>c</sup>Johansen 'Maximal Eigenvalue test'.

<sup>d</sup>Systems of spot exchange rates and one month interest rates for dollar and foreign currency, together with one month forward interest rates for dollar and foreign currency.

<sup>e</sup>Significant at the 90% confidence level.

<sup>f</sup>Significant at the 95% confidence level.

<sup>g</sup>Systems of spot exchange rates and three month interest rates for dollar and foreign currency, together with three month forward interest rates for dollar and foreign currency.

### 3. Explaining the additional trends

#### 3.1. Infrequent shifts in spot rate process and additional trends

The results above suggest there are trends in forward rates additional to those in spot rates. In this section, we will show that shifts in the process followed by spot rates can make excess returns appear to contain additional trends.<sup>8</sup> These shifts may result from switches in the process of fundamentals driving spot rates, or from the presence of heterogeneous traders in the market.

To make this explanation simple, suppose that market expectations of future spot rates depend upon two possible processes,  $(s_t|j)$  for  $j=A, B$ . The expected future spot rate is

$$E_t s_{t+k} = (1 - \lambda_t) E_t(s_{t+k}|A) + \lambda_t E_t(s_{t+k}|B), \quad (5)$$

where  $E_t$  is the expectations operator conditional upon information available at time  $t$ , and  $\lambda_t$  is the probability at time  $t$  that the spot rate will follow the  $B$  process at  $t+k$ .

There are at least two ways in which market expectations could take the form of eq. (5). The first is that the fundamentals process generating the spot

<sup>8</sup>Both explanations are discussed in greater detail in Evans and Lewis (1992).

rate may change infrequently. For example, the behavior of monetary or fiscal policy may undergo regime changes. If expectations of these events are anticipated rationally, then market expectations will take the form of (5) where  $(s_t|A)$  denotes the process currently generating the spot rate and  $(s_t|B)$  denotes the anticipated process of the spot rate in the event that the policy regime shifts. In this case,  $\lambda_t$  is the probability of the regime shift from time  $t$  to  $t+k$ .

Research on the effects of heterogeneous traders by Cutler et al. (1990), and Frankel and Froot (1986) suggests another reason why market expectations may take the form of (5). These studies argue that some traders are rational and informed while others chase trend movements in asset prices. Even though rational traders are in the market, risk aversion limits the trades that they are willing to take against other less-informed traders. As a result, the price may trend away from its fundamental value for significant periods of time.

Suppose that rational informed traders know that the price will eventually revert to its level implied by the fundamentals process. Defining this spot price process as  $(s_t|B)$  and the process following the current trend as  $(s_t|A)$ , the rational traders' forecast of the future price is given by (5). In this case,  $\lambda_t$  is the rational traders' assessed probability that the price process will revert to its fundamental level between  $t$  and  $t+k$ .

Regardless of whether heterogeneous traders or regime shifts are responsible, expectations in the form of eq. (5) can induce additional trends in the relationship between spot and forward rates during periods when the shifts do not occur. To see why, we write the spot rate processes in each regime in terms of permanent and transitory components:

$$(s_t|j) = u_{j,t} + e_{j,t}, \quad u_{j,t} = u_{j,t-1} + \eta_{j,t}, \quad j = A, B, \tag{6}$$

where  $u_{j,t}$  is independent and identically distributed, and  $e_{j,t}$  is a stationary I(0) process. Substituting (6) into (5), and combining the result with (1) implies

$$f_{t,k} = E_t s_{t+k} + \text{I(0) terms} = (1 - \lambda_t) \sum_{\tau=1}^t \eta_{A,\tau} + \lambda_t \sum_{\tau=1}^t \eta_{B,\tau} + \text{I(0) terms}. \tag{7}$$

Consider the implications of (6) and (7) when shifts in the process of spot rates occur infrequently. Between shifts the spot rate will be drawn from a given process, say process  $A$ . During these periods, the spot rate will be driven only by permanent shocks to process  $A$ ,  $\eta_{A,\tau}$ , while the forward rate will be driven by both  $\eta_{A,\tau}$  and  $\eta_{B,\tau}$  (provided  $\lambda_t > 0$ ). As a result, forward rates will appear to contain an additional trend compared to the current spot rate.

### 3.2. Evidence from a Monte Carlo example

To investigate whether infrequent shifts in the process of spot rates could be responsible for our results above, we conducted a Monte Carlo experiment. We calibrated our experiment using the Markov-switching model for the spot dollar/pound exchange rate process estimated by Engel and Hamilton (1990), modified to be consistent with the expectations in (5).<sup>9</sup> For each experiment we generated 15 years of quarterly observations for the spot exchange rate from

$$(s_t|j) = (s_{t-1}|j) + \eta_{j,t}, \quad j = A, B, \quad (8)$$

with  $\text{Var}(\eta_{A,t}) = 16.92$ ,  $\text{Var}(\eta_{B,t}) = 20.25$ . Next, we use (7) to calculate the corresponding forward rate as  $f_{t,1} = E_t s_{t+1}$ . These forward rates include the effects of anticipated switches in the spot rate through the transition probabilities  $\lambda$ .<sup>10</sup> We then calculated the two Johansen test for these generated forward and spot rates. This procedure was repeated 1,000 times.

Table 4 reports the results of these experiments for the transition probabilities ranging from 0.50 to 0.95. The top panel shows the information about the distribution of the Trace and Eigenvalue tests. The columns labeled 'mean' provide the mean of the distribution while the columns labeled 'size' report the probability of observing values of the statistics greater than the 95% critical level of the Johansen distribution. Since our experiments allow the spot rate to switch between processes, and forward rates will only appear to contain an additional trend between switches, (by construction) forward and spot rates ought to contain the same number of trends in long data samples. In other words, given a long enough sample, we should expect to reject the null hypothesis of two stochastic trends 95% of the time. Our experiments based on 15 year samples show that the probabilities are significantly less than 95%. The probabilities of rejecting range from 0.849 for the Eigenvalue test when the transition probability is 0.95 to 0.330 for the Trace test when the transition probability is 0.70. These results show that in typical samples there is a reasonably high probability of observing an additional trend in forward rates when traders expect shifts in the spot process.

These results seem to imply that trends should be apparent in excess returns  $s_{t+1} - f_{t,1}$ . However, as the lower panel of table 4 shows, the

<sup>9</sup>Engel and Hamilton (1990) assume that shifts of the process correspond to the dynamics of the exchange rate alone. We allow for alternative trends between spot and forward rates since the entire process shifts between the two states in (8). Note that a shift in the process will imply a discrete jump from  $(s_t|A)$  to  $(s_t|B)$  or vice versa.

<sup>10</sup>We ignored the  $I(0)$  terms in (7) for simplicity. This should not affect the number of trends in spot and forward rates.

Table 4  
Simulation results.<sup>a</sup>

<i>Tests for two trends in spot and one month forward rates</i>				
Transition probability	$T_1^b$		$T_2^c$	
	Mean <sup>d</sup>	95% size <sup>e</sup>	Mean	95% size
0.95	25.297	0.793	23.877	0.849
0.90	20.994	0.635	19.521	0.727
0.80	16.546	0.382	15.245	0.487
0.70	15.919	0.330	14.776	0.466
0.60	16.638	0.381	15.529	0.504
0.50	18.062	0.495	16.941	0.613

<i>Summary statistics on forward premia</i>				
Transition probability	Mean (Std. dev.)	$\rho_1$ (Std. dev.)	$\rho_6$ (Std. dev.)	$\rho_{12}$ (Std. dev.)
0.95	0.035 (1.606)	0.114 (0.162)	0.091 (0.151)	0.079 (0.140)
0.90	0.044 (2.531)	0.210 (0.188)	0.178 (0.180)	0.142 (0.163)
0.80	0.053 (3.862)	0.202 (0.184)	0.209 (0.163)	0.156 (0.156)
0.70	0.079 (4.567)	0.055 (0.193)	0.166 (0.151)	0.123 (0.134)
0.60	0.085 (4.818)	-0.121 (0.197)	0.118 (0.152)	0.081 (0.127)
0.50	0.074 (4.667)	-0.291 (0.192)	0.092 (0.164)	0.046 (0.142)

<sup>a</sup>The spot exchange rate process is assumed to follow  $\Delta x_t = u_t^i$ , where  $\text{Var}(u_t^1) = 16.92$  for State 1 and  $\text{Var}(u_t^2) = 20.25$  for State 2. [The variance parameters are from Engel and Hamilton (1990).] Forward rates are calculated as the expected future spot rates,  $f_t^i = E_t x_{t+1}$  using the transition probabilities that show the probability of remaining in state  $i$  ( $i=1$  and  $2$ ) between  $t$  and  $t+1$ .

<sup>b</sup>Johansen 'Trace test' for two trends in  $(x_t, f_t^i)$ .

<sup>c</sup>Johansen 'Maximal Eigenvalue test' for two trends in  $(x_t, f_t^i)$ .

<sup>d</sup>Mean value of statistics in 1,000 replications.

<sup>e</sup>Probability of observing a value for  $T_i$  greater than the 95% confidence level.

unconditional mean and the autocorrelation coefficients are close to zero in all of our experiments. The reason is that the trend component in excess returns generated by our experiments is very small.<sup>11</sup> This explains why standard tests have failed to find evidence of trends in excess returns before. These statistics also show that the serial correlation in excess returns

<sup>11</sup>Monte Carlo studies of time series with small unit roots suggest that they are more appropriately treated as stationary for econometric purposes [see Campbell and Perron (1991)]. Thus, our explanations for the presence of additional trends should *not* be viewed as a reason for treating excess returns as non-stationary variables.

generated by our experiments are similar to those in actual bond and foreign exchange returns.

#### 4. Concluding remarks

In this paper, we tested the restrictions implied by the conventional view that spot and forward rates follow processes with permanent disturbances while excess returns are stationary. Surprisingly, we found that the restrictions are rejected for spot and forward exchange rates (in the case of the pound/dollar and yen/dollar rates) and for several Eurocurrency rates. A number of Monte Carlo experiments showed these results to be quite robust, so that excess returns in a number of markets contain trends.

We then considered possible explanations for these findings. We showed that if expectations incorporate expectations of shifts in the process of spot rates, then we would be likely to find evidence of additional trends in excess returns, even though these trends would disappear in an infinite sample. Our findings suggest that future research should examine the sources of trends in excess returns.

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