On-Line Appendices: How Can Asset Prices Value Exchange Rate Wedges?

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August 23, 2022

A Data and Simulated Method of Moments

In this appendix, we describe the data measurement, Simulated Method of Moments (SMM) approach used in Table 1 and the implied consumption processes for the two-country example in Tables 2, 3, 4, and 5. Appendix D.1 describes the data moments used in the SMM extension to multiple countries.

A.1 Data Measurement

The welfare costs measurement requires real consumption data for each country. Feenstra, Inklaar, and Timmer (2015) describe issues related to the comparability of prices and consumption over countries and time, arguing that welfare valuations require measures that allow direct international comparison. We therefore follow their suggested comparison, using consistent real expenditure consumption series and prices in the Penn World Tables (PWT) Version 9.1 released in 2021. This version provides data on consumption and local prices as well as prices against a US numeraire through 2017.[1] The data series measure the relative price of goods indices in non-US countries relative to the value of the US con-

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[1] Alternative data sources such as the PWT output data and price are described in Appendix D.2.

sumption goods index measured in US dollars at the base year. The consumption for each country j is measured in local goods prices, denoted here as $\hat{P}^j$. However, for international comparison, the relative prices differ due to exchange rate variations. Then, for international comparison, we use real domestic consumption measured in units of the U.S. as numeraire. That is: $\widetilde{C}^{j,D}_{t+1} = S^j_{t+1} C^j_{t+1}$ where $S^j_{t+1} = e^j_t \hat{P}^j_t / P^J_t$ where J is the US (numeraire country), and e^j_t converts the nominal value of country j into US dollars. All prices and exchange rates are measured relative to the index in the base year. In the text, we use direct measures of real consumption in local prices, C^j_t, and real exchange rates across countries involving international prices. Therefore, we subsume the nominal exchange rate in the definition of the real exchange rate measure used in the international price numbers. That is, $S^j_{t+1} = e^j_t \hat{P}^j_t / P^J_t = P^j_t / P^J_t$ where P^j measures the prices level of country j at numeraire prices. Moreover, in our two country example, the total number of countries is 2 so that we ignore j for parsimony until we consider the multi-country extension in Section 4 and Appendix D.1.

Our analysis also requires data for asset returns and dividends. For risk-free rates, we use the annual average 90 day government bill rate from the OECD. For equity returns and dividends, quarterly data from the Total Market Indices in Datastream-Thomson Financial are targeted. Nominal equity returns are computed from 1970 to 2018, while dividend growth data moments are computed up to 2009. To match the annual level consumption data, we deflate nominal equity returns and dividend growth with the same consistent price deflators from the Penn World Tables. These real annual equity returns, risk-free rates, and dividend growth rates provide asset return moments to be matched by the simulated method of moments described below.

A.2 Simulated Method of Moments Approach

To solve for the consumption process parameters used in Table 1, we fit key target annual moments from a reduced SMM analysis described next.

A.2.1 Consumption Processes

We begin by defining the consumption growth process measured in the data as: $g_{c,t+1} \equiv \ln(C^D_{t+1}/C^D_t)$ for the domestic country, and $g_{\widetilde{c},t+1} \equiv \ln(\widetilde{C}^D_{t+1}/\widetilde{C}^D_t)$ for the foreign country. For the two country example, these countries follow symmetric processes and therefore we describe only the process for the domestic country for parsimony.

For the i.i.d. consumption version, this consumption process is given by:

$$g_{c,t+1} = \mu + \sigma\nu_{t+1} \tag{A.1}$$

where $\nu_{t+1} \backsim N(0,1)$ and where the correlation between the two countries is given by $Corr(\nu_{t+1}, \widetilde{\nu}_{t+1})$.

Alternatively, for the persistent consumption version we allow for an autoregressive "long run risk" processes as in:

$$\begin{aligned} g_{c,t+1} &= \mu + x_t + \sigma\nu_{t+1} \\ x_{t+1} &= \rho x_t + \varphi_e \sigma e_{t+1} \end{aligned} \tag{A.2}$$

where $\nu_{t+1}, e_{t+1} \backsim N(0,1)$ and where ν_{t+1}, e_{t+1} are mutually independent. To fit the asset return series in the simulation, we substitute these candidate consumption processes into the Euler equation in order to value the returns. We choose these processes due to their simplicity and common usage only. For example, as described in Section 4, other candidate processes can be used such as disaster risk and habit persistence.

A.2.2 Asset Returns and Consumption

In order to match the asset return moments to consumption as reported in Table 1, we require the asset return solutions for equity and for the risk-free rate. For the i.i.d. case, these solutions are provided in classic references such as Mehra and Prescott (1985), while the asset return solutions for the consumption process in equation (A.2) are detailed in Bansal and Yaron (2004). Below we detail the solutions for the persistent consumption process in equation (A.2) only since the corresponding solution for the i.i.d. version in equation (A.1) is nested by assuming the persistent consumption variance term, $\varphi_e = 0$.

The stochastic discount factor (SDF) is the underlying variable that values asset returns. Under Constant Relative Risk Aversion preferences, this SDF can be measured directly with consumption growth as: $(C_{t+1}/C_t)^{-\gamma}$. However, more generally, with recursive preferences, this SDF is determined by the Euler equation that values the claim on lifetime consumption; i.e. the return on wealth. The return on this process as given by consumption in the data as:

$$R^D_{c,t+1} \equiv \left(\frac{C^D_{t+1}}{C^D_t}\right)\left(\frac{1+Z^D_{t+1}}{Z^D_t}\right) \tag{A.3}$$

where Z^D_t is the ratio of the time t value of an asset paying out the consumption process divided by current consumption. The Campbell-Shiller approximation then implies that the natural logarithm of this return, $r^D_c \equiv \ln R^D_c$ is a function of the logarithm of the value of wealth-to-consumption ratio, $z^D_t \equiv \ln(Z^D_t)$, and of consumption growth, $g^D_{c,t+1}$, specified by:

$$r^D_{c,t+1} = k_0 + k_1 z^D_{t+1} - z^D_t + g^D_{c,t+1} \tag{A.4}$$

where k_0 and k_1 represent approximating constants.[2]

The SDF determined by this return on wealth then provides a valuation of equity as a dividend-paying asset. Defining the price-to-dividend ratio as: $z_{m,t} \equiv \ln(P_t/D_t)$, and dividend growth as $g_{d,t+1} \equiv \ln(D_{t+1}/D_t)$, the log equity returns follow:

$$r^D_{m,t+1} = k_{m,0} + k_{m,1} z^D_{m,t+1} - z^D_{m,t} + g^D_{d,t+1} \tag{A.5}$$

where $k_{m,0}$ and $k_{m,1}$ are approximating constants based upon the Campbell-Shiller approximation. For this process, we specify the process as:

$$g_{d,t+1} = \mu_d + \phi x_t + \varphi_d \sigma \nu_{t+1} \tag{A.6}$$

The risk-free rate, $R^D_{f,t}$, is the return on an asset purchased at time t that pays one unit of domestic consumption with certainty at $t+1$. To value this return, we again follow the

[2]The constants are $k_1 = \frac{exp(\bar{z})}{1+exp(\bar{z})}$ and $k_0 = log(1+exp(\bar{z})) - k_1\bar{z}$, where $\bar{z}$ is the steady state log price to consumption ratio.

literature to apply the Euler equation of the domestic agent to value this risk-free rate.[3] Since the risk-free rate is known at time t, the Euler equation can be written as:

$$E_t\left\{M^D_{t+1}\right\} R_{f,t} = 1 \tag{A.7}$$

So far, these asset returns are given as general solutions that do not rely on preferences. However, to conduct SMM on these returns, we require a view on preferences. Epstein-Zin utility provides a convenient benchmark for these purposes since it maps directly to many of the approaches in the literature cited in the text. Under these preferences, the SDF measured with data is:

$$M^D_{t+1} \equiv \beta^\theta (C^D_{t+1}/C^D_t)^{\left(-\frac{\theta}{\psi}\right)} (R^D_{c,t+1})^{(\theta-1)} \tag{A.8}$$

where $\theta = (1-\gamma)/(1-\psi^{-1})$ and where R^D_c is determined by the Z^D_t in equation (A.3) that solves for the value of the wealth return:

$$E_t\left\{\beta^\theta (C^D_{t+1}/C^D_t)^{\left(-\frac{\theta}{\psi}\right)} (R^D_{c,t+1})^{\theta}\right\} = 1 \tag{A.9}$$

Therefore, in our analysis below, we will require measures of only three preference parameters: intertemporal elasticity of substitution (IES) given by ψ, the relative risk aversion coefficient γ, and the time discount factor β. Throughout the paper, we assume these parameters are the same across countries.

A.2.3 Simulations Approach

Matching the consumption processes to asset returns requires fitting the consumption payout process in the wealth return equation (A.4) jointly with the asset return equations (A.5) and (A.7). We begin with preference estimates that have been found to fit asset returns best in the US data from Bansal and Yaron (2004). Specifically, we take the parameters of $\psi = 1.5$, $\gamma = 10$, and annualized $\beta = .985$ or monthly $\beta = .998$. The difference in β depends upon

[3] As described in the text, if markets are incomplete then at most one investor's Euler equation will be marginal in the data. Here we follow the standard approach in the consumption-based asset pricing literature that the domestic agent is the marginal investor for domestic assets. For an alternative assumption that assumes foreigners price the domestic risk-free rate, see Lustig and Verdelhan (2019).

whether we simulate the model at the annual level or simulate at the monthly level and then time-aggregate to annual moments. For our example in Table 1, we use monthly data to provide the best fit, although throughout the rest of the paper we alternatively use an annual model following Colacito and Croce (2013).

To generate the parameter values, we first calibrate the monthly growth rates μ and μ_d to the annual means of consumption growth and dividend growth. To simplify, we calculate the mean annual growth rates from the data and divide by 12. In trial runs of the SMM procedure described below, this calibration assumption makes little difference in the estimates of the remaining parameters but greatly decreases the computation time.

We then use the reduced SMM to fit the remaining parameters for each country:$[\sigma, \varphi_e, \rho, \phi, \varphi_d]$ to match dividend behavior for the equity return in equation (A.5). Implementing the SMM procedure involves two main steps. First, for every set of parameter values, solve the model using the analytical solutions for returns using a set of targeted moments to best represent both consumption and asset pricing data.[4] Second, compute a weighted difference between the targeted set of model-generated moments and the data moments using a weighting matrix. To treat all targets equally, we report the estimates using the identity matrix.[5] The set of parameter values that minimizes this difference is the SMM estimate.

For the time aggregated case, we compute the growth rate between the levels at t and $t+12$, given the realizations of 12 monthly growth rates.[6] To match our annual consumption, dividend growth and asset return moments, we then time-aggregate the model-generated data from monthly to annual frequency. Parameter estimates and simulated model moments are the averages of 500 simulations, each with 840 time-aggregated monthly observations.

For the asset return model, we choose the following set of target data moments for each country as reported in Table 1: the standard deviation of log consumption growth, the first order auto-correlation of log consumption growth, the mean equity premium, the mean risk-free rate, the standard deviation of the market return, and the standard deviation of the risk-free rate. Using these six moments per country, we estimate the three consumption growth

[4]See Gallant and Tauchen (1999) for a discussion on efficient method of moments and problems with moment selection.

[5]We also implemented the reduced SMM procedure using a diagonal matrix with typical components equal to the sample variance. This procedure gave qualitatively similar results.

[6]By comparison, we multiply monthly rates times 12 when we annualize as opposed to time aggregate.

parameters capturing the transitory risk, σ, persistent risk, φ_e, and degree of persistence, ρ. As a practical matter, we find that fitted values of ρ are quite similar across countries so we equate them in the analysis.

To accommodate dividend growth in the model, we augment the six moments basic moments described above to include the standard deviation of dividend growth, and the first order auto-correlation of log dividend growth. We therefore include the standard deviation of real dividend growth of 13.01% and 13.6% for Canada and the United States respectively, as well as the first order auto-correlation of log dividend growth of 0.45 for Canada and 0.08 for the United States. Using these eight moments per country, we fit the same three consumption parameters along with the two new parameters, the relative volatility of monthly dividend growth φ_d, and the consumption leverage parameter, ϕ. In fitting these two parameters, we impose bounds from the literature. Specifically, Abel (1999) suggests the leverage parameter can be guided by the ratio of the standard deviation of dividend growth over that of two variables: income growth and consumption growth. These measures calibrated in Abel (1999) using U.S. data suggested a range of $\phi \in \{2.5, 3.6\}$. For comparison, we also consider a version of the model restricting $\phi = 3$ for all countries. Similarly, we impose a range on $\varphi_d \in \{1, 6\}$ allowing for significantly higher volatility in dividends than consumption, which includes the calibrated value of $\varphi_d = 4.5$ in Bansal and Yaron (2004).

Table 1C shows the results of this fit. The mean estimated parameters from the Simulated Method of Moments are: $[\rho = 0.988, \sigma = 0.0074, \phi = 3.4, \phi_e = 0.0387, \phi_d = 5.8]$ for Canada and $[\rho = 0.986, \sigma = 0.0068, \phi = 2.9, \phi_e = 0.0448, \phi_d = 5.95]$ for the Untied States. Both sets of parameters are fairly consistent with previous estimates and calibration in the literature. While the variability of asset returns are generally lower than in the data, the match can be improved by introducing time-varying volatility as in Wachter (2013) and Bansal and Yaron (2004). We instead maintain this more parsimonious model to illustrate how asset returns can be used to discipline the valuation of wedges.

A.3 Cross Country Moments

The SMM approach described above disciplines the consumption process within countries based upon a common assumption in the literature that domestic investors price their own assets. Notably, we did not target any of the cross-country moments. However, valuing the cross-country valuations requires assumptions about the international correlations of consumption and exchange rates.

For the i.i.d consumption growth examples in Tables 1 to 4, the cross-country correlations are given by $Corr(\nu_{t+1}, \widetilde{\nu}_{t+1})$ as specified following equation (A.1). We calibrate these moments with the simple correlations of consumption growths across countries. Thus, for the US and Canada in Table 1A, this correlation is calibrated in the observed data to be 0.574. To consider how exchange rate variability impacts the value of foreign wealth returns, we also require a measure of the correlation between consumption and exchange rates in the data. Assuming the exchange rate growth is also i.i.d., this growth follows a process:

$$g_{s,t+1} = \ln(S^D_{t+1}/S^D_t) = \sigma_s \nu^s_{t+1} \tag{A.10}$$

where $\nu^s_t \backsim N(0, 1)$ and where the correlation between domestic consumption growth and the exchange rate is given by $Corr(\nu_{t+1}, \nu^s_{t+1})$. We calibrate this value to the measure in the data of -0.18. Note that, as pointed out by Backus and Smith (1993), the consumption and exchange rate growth rates are relatively uncorrelated.

The case of persistent consumption growth in equation (A.2) additionally requires an assumption about the correlation of the persistent "long run risk" component, e_t. Colacito and Croce (2011) show that a correlation near one matches exchange rates under complete markets. Furthermore, Lewis and Liu (2015) show that a high correlation across countries is required to simultaneously explain the high correlations of asset returns but more modest correlation of consumption. They also show that the gains from international diversification are minimized when the persistent risk correlation is high. Therefore, to provide a conservative lower bound on the costs of wedges in our persistent risk examples, we assume that $Corr(e_t, \widetilde{e}_t) = 1$ throughout the text. Appendix D.2 describes more general versions of the exchange rate process in which this correlation may be more important.

In addition, while we do not target the variability of the foreign exchange risk premium, this return is implied by the returns. For example, the return on borrowing at the domestic rate, investing in the foreign risk free rate and converting back into domestic consumption units at the end of the period is:

$$g_{fx,t+1} = \ln(S^D_{t+1}/S^D_t) + \widetilde{r}_{f,t} - r_{f,t} \tag{A.11}$$

This process therefore includes both the volatility in the exchange rates and the risk-free rates implied by our SMM results above.

B Wedge Valuation Details

This appendix summarizes the wedge valuations.

B.1 Calculating the Value of the η Wedge

As discussed in the text, valuing the η Wedge requires calculating the price ratio for the foreign investor valuation of the domestic wealth return payouts as inferred by consumption data. To illustrate, we take the Euler Equation of the foreign investor for this identification.[7] In this case, the valuation to the foreign investor of the domestic wealth return is given by:

$$E_t \left\{ \widetilde{M}^D_{t+1} R^D_{\eta,t+1} \left(S^D_{t+1}/S^D_t \right)^{-1} \right\} = 1 \tag{B.1}$$

where S^D_{t+1}/S^D_t is the change in exchange rate that converts the domestic consumption payouts into foreign goods and $R_{\eta,t+1}$ is the return on a claim that domestic consumption, valued by the foreign investor. That is, the return is:

$$R^D_{\eta,t+1} = (C^D_{t+1}/C^D_t)(1 + Z^\eta_{t+1})/Z^\eta_t. \tag{B.2}$$

where Z^η_t is the price that solves the Euler equation (B.1) given the domestic consumption process. Substituting the stochastic discount factor, $\widetilde{M}^D_t$, implied by Epstein-Zin-Weil

[7] A symmetric relationship holds for the domestic investor valuation of claimes on lifetime foreign consumption.

preferences given in equation (A.8) into equation (B.1) gives:

$$E_t\left\{\beta^{\theta}(\widetilde{C}^{D}_{t+1}/\widetilde{C}^{D}_{t})^{(-\frac{\theta}{\psi})}(\widetilde{R}^{D}_{c,t+1})^{\theta-1}R^{D}_{\eta,t+1}(S^{D}_{t+1}/S^{D}_{t})^{-1}\right\}=1. \tag{B.3}$$

Note that the return on domestic wealth to the foreign investor, R_η differs from the return to the domestic investor, R_c, due to the difference in price ratio of Z^η in equation (B.2) versus Z^D. Nevertheless, since they both are claims on domestic consumption, for expositional simplicity we denote both measures of consumption assets as R_c in the text with some abuse of notation.

To solve this Euler equation, we assume joint-log normality following much of the literature, as well as the SMM identification in Appendix A. Then, taking logs of both sides of equation (B.3), and using the notation $g_{b,t}$ to refer to the time t log growth rate of any variable b implies:

$$E_t[\theta\ln\beta-\frac{\theta}{\psi}\tilde{g}^{D}_{c,t+1}+(\theta-1)\tilde{r}^{D}_{c,t+1}+r^{D}_{c,t+1}-g^{D}_{s,t+1}]+\frac{1}{2}Var[\theta\ln\beta-\frac{\theta}{\psi}\tilde{g}^{D}_{c,t+1}+(\theta-1)\tilde{r}^{D}_{c,t+1}+r^{D}_{c,t+1}-g^{D}_{s,t+1}]=0 \tag{B.4}$$

We consider two examples of consumption growth: (1) i.i.d.; and (2) persistent long run risk.

(1) i.i.d consumption For the i.i.d. symmetric consumption case, each country has identical consumption processes given by equation (A.1). This assumption treats the processes of each country with their own random walk component. Alternatively, these processes could include a cointegrating error-correction term as in Colacito and Croce (2013). We discuss this possibility in Appendix D.2.

Furthermore, we require measurement of prices in the data in order to value relative consumption expenditures. For this purpose, we assume that the exchange rate in the data also follows the random walk process as in equation (A.10), although Appendix D also describes alternative exchange rate processes that allow for long term mean reversion in the real exchange rate. To illustrate the random walk base case, however, substituting equations (A.1) and (A.10) into equation (B.4) and solving for Z^η gives the wealth price for this investor.

To compare the value of the same lifetime consumption process to the domestic investor, we follow similar steps but use the domestic wealth return Euler equation.

$$E_t\left\{M^D_{t+1}R^D_{c,t+1}\right\} = 1 \tag{B.5}$$

Solving this equation for Z^D provides the comparison to calculate the η wedge in the text, repeated here.

$$(1-\Delta_{D,\eta}) = \frac{V(W^D/C^D)}{V(W^\eta/C^\eta)} = \left\{\frac{1+Z^D}{1+Z^\eta}\right\}^\Psi \tag{B.6}$$

(2) With persistent risk: For persistent consumption, we repeat these steps but assume instead that the consumption process includes a persistent long run risk component given by equation (A.2). To value the wealth series, we therefore substitute this process for $g^D_{c,t+1}$ into the Euler equations (B.4) as well as the log counterpart to equation (B.5). In this persistent consumption case, the price ratios, Z^D and Z^η, are time varying.. Therefore, we follow the same approach as described in the SMM analysis of Appendix A. That is, we assume $z_{\eta,t} = \bar{A}_{\eta,0} + \bar{A}_{\eta,1}x_t + \bar{A}_{\eta,2}\tilde{x}_t$ and use the Campbell-Shiller approximation to solve returns as in equation (A.4). This assumption applied to the two Euler equations provides the measure of Z^D and Z^η needed to calculate the certainty equivalent measure $\Delta_{D,\eta}$ as above.

B.2 Calculating the Value of the Total ζ Wedge

In order to calculate the value of the total deviation from complete markets, $\Delta_{D,*}$, we follow similar steps to solve for the Euler equations as above. First, we use the solution for the price ratio Z^D as described in B.1, using Euler equation (B.5). Second, we require the solution to the complete markets consumption process depending on the view of the exchange rate determination described in Appendix C. Using this solution, we follow the same steps to solve for Z^* for returns in the Euler equation given by:

$$E_t\left\{\beta^\theta (C^*_{t+1}/C^*_t)^{\left(-\frac{\theta}{\psi}\right)}(R^*_{c,t+1})^\theta\right\} = 1. \tag{B.7}$$

The wealth ratio Z^* that solves for this equation provides the comparison for calculating the total cost $\Delta_{D,*}$ in the text, repeated here.

$$1 - \Delta_{D,*} = \left\{ \frac{1 + Z^D}{1 + Z^*} \right\}^{\Psi}$$

Appendix C below describes the solutions to these complete markets price ratios, Z^*, with each depending upon exchange rate views.

B.3 Calculating the Value of the Total S Wedge

We next describe the relationship between wealth returns under complete markets as well as its implied "Total" exchange rate wedge.

B.3.1 Complete Markets Wealth Returns Relationship

To see the connection between wealth returns under complete markets, note that these returns can be rewritten as a sharing rule for aggregate world resources in numeraire prices, denoted Y_t^w. In this case, the complete markets wealth return for the domestic economy can be written:

$$\begin{aligned} R^*_{c,t+1} &= \left(\frac{C^*_{t+1}}{C^*_t} \right) \left(\frac{1 + Z^*_{t+1}}{Z^*_t} \right) \\ &= \left(\frac{\omega_{t+1} Y^w_{t+1}}{\omega_t Y^w_t} \right) \left(\frac{S^*_{t+1}}{S^*_t} \right)^{-1} \left(\frac{1 + Z^*_{t+1}}{Z^*_t} \right) \end{aligned} \tag{B.8}$$

where, as in the text, ω is the share of world resources consumed by the domestic agent, S^* is the exchange rate, and Z^* is the wealth price-to-consumption ratio, all implied by the complete markets allocation. Similarly, for the foreign numeraire country, the wealth return is:

$$\begin{aligned} \widetilde{R}^*_{c,t+1} &= \left(\frac{\widetilde{C}^*_{t+1}}{\widetilde{C}^*_t} \right) \left(\frac{1 + \widetilde{Z}^*_{t+1}}{\widetilde{Z}^*_t} \right) \\ &= \left(\frac{\widetilde{\omega}_{t+1} Y^w_{t+1}}{\widetilde{\omega}_t Y^w_t} \right) \left(\frac{1 + \widetilde{Z}^*_{t+1}}{\widetilde{Z}^*_t} \right) \end{aligned} \tag{B.9}$$

where $\widetilde{\omega}$ is the share of resources consumed by the foreign household.

Next, combine this relationship with the fact that price-adjusted stochastic discount factors are equalized as given in equation (1) in the text, repeated here:

$$M^*_{t+1}\left(S^*_{t+1}/S^*_t\right) = \widetilde{M}^*_{t+1} \ . \tag{B.10}$$

Recall that the stochastic discount factor for Epstein-Zin preferences is:

$$\beta^\theta (C_{t+1}/C_t)^{\left(-\frac{\theta}{\psi}\right)} (R_{c,t+1})^{(\theta-1)} \ . \tag{B.11}$$

Then substituting this expression into equation (B.10) implies:

$$\beta^\theta (C^*_{t+1}/C^*_t)^{\left(-\frac{\theta}{\psi}\right)} (R^*_{c,t+1})^{(\theta-1)} \left(S^*_{t+1}/S^*_t\right) = \beta^\theta (\widetilde{C}^*_{t+1}/\widetilde{C}^*_t)^{\left(-\frac{\theta}{\psi}\right)} (\widetilde{R}^*_{c,t+1})^{(\theta-1)} \tag{B.12}$$

or, using the risk-sharing share of global resources yields:

$$\beta^\theta \left(\frac{\omega_{t+1}Y^w_{t+1}}{\omega_t Y^w_t}\right)^{\left(-\frac{\theta}{\psi}\right)} (R^*_{c,t+1})^{(\theta-1)} \left(\frac{S^*_{t+1}}{S^*_t}\right) = \beta^\theta \left(\frac{\widetilde{\omega}_{t+1}Y^w_{t+1}}{\widetilde{\omega}_t Y^w_t}\right)^{\left(-\frac{\theta}{\psi}\right)} (\widetilde{R}^*_{c,t+1})^{(\theta-1)} \tag{B.13}$$

Then, substituting the domestic and foreign wealth returns in equations (B.8) and (B.9) into equation (B.13), and using the fact that $\omega_t + \widetilde{\omega}_t = 1$ implies:

$$\frac{\widetilde{\omega}_{t+1}}{\widetilde{\omega}_t} = \left[1 + \left(\frac{S^*_{t+1}}{S^*_t}\right)^{1-\frac{1}{\gamma}} \left(\frac{(1+\widetilde{Z}^*_{t+1})/\widetilde{Z}^*_t}{(1+Z^*_{t+1})/Z^*_t}\right)^{\left(\frac{1-\theta}{\gamma}\right)}\right] \tag{B.14}$$

Substituting these solutions for the growth rate in shares implies:

$$M^*_{t+1}R^*_{c,t+1} = \widetilde{M}^*_{t+1}\widetilde{R}^*_{c,t+1}\left(S^*_{t+1}/S^*_t\right)^{-(1-\frac{1}{\gamma})} \left(\frac{(1+Z_{t+1})/Z_t}{(1+\widetilde{Z}_{t+1})/\widetilde{Z}_t}\right)^{\frac{1}{\gamma}\frac{\theta}{\psi}} \tag{B.15}$$

When the consumption process is i.i.d., then $Z_t = Z$, a constant for all t. Thus, this relationship becomes:

$$M^*_{t+1}R^*_{c,t+1} = \widetilde{M}^*_{t+1}\widetilde{R}^*_{c,t+1}\left(S^*_{t+1}/S^*_t\right)^{-(1-\frac{1}{\gamma})}\left(\frac{(1+Z)/Z}{(1+\widetilde{Z})/\widetilde{Z}}\right)^{\frac{1}{\gamma}\frac{\theta}{\psi}} \tag{B.16}$$

Furthermore, when the countries are symmetric in local price units, then $Z = \widetilde{Z}$ and therefore,

$$M^*_{t+1}R^*_{c,t+1} = \widetilde{M}^*_{t+1}\widetilde{R}^*_{c,t+1}\left(S^*_{t+1}/S^*_t\right)^{-(1-\frac{1}{\gamma})} \tag{B.17}$$

B.3.2 Total S-Wedge Decomposition

We can then write the "Total" S-Wedge as the wedge deviation between the state price of wealth returns under complete markets, distorted only by the exchange rate in the data as:

$$M^*_{t+1}R^*_{c,t+1} = \widetilde{M}^*_{t+1}\widetilde{R}^*_{c,t+1}\left(S^D_{t+1}/S^D_t\right)^{-(1-\frac{1}{\gamma})}\left(\zeta_{s,t+1}\right)^{-(1-\frac{1}{\gamma})} \tag{B.18}$$

Relating this relationship to all of the data-inferred measures of wealth implies:

$$M^*_{t+1}R^*_{c,t+1} = \widetilde{M}^D_{t+1}\widetilde{R}^D_{c,t+1}\left(S^D_{t+1}/S^D_t\right)^{-(1-\frac{1}{\gamma})}\left(\eta^T_{t+1}\right) \tag{B.19}$$

Thus, for symmetric countries, this η^T wedge measures the difference in valuations between complete markets and wealth in the data recognizing that countries value their lifetime consumption with their own stochastic discount factors, but differ in exchange rate valuations only.

B.4 Summary of Wedge Decompositions

This approach can be used more generally to value other distortions away from complete markets.

B.4.1 General Wedges Decomposed

As described in the discussion above, we can value various deviations from complete markets using this framework. For example, we can decompose the components of the ζ wedge in local price units into proportional "wedges" through the following identities:

$$\begin{aligned} M_{t+1}^{D} &\equiv \zeta_{M,t+1} M_{t+1}^{*};\ \widetilde{M}_{t+1}^{D} \equiv \zeta_{\widetilde{M},t+1} \widetilde{M}_{t+1}^{*} \\ \left(\frac{S_{t+1}^{D}}{S_{t}^{D}}\right) &\equiv \zeta_{S,t+1} \left(\frac{S_{t+1}^{*}}{S_{t}^{*}}\right) \\ R_{a,t+1}^{D} &\equiv \zeta_{R_a,t+1} R_{a,t+1}^{*};\ \widetilde{R}_{a,t+1}^{D} \equiv \widetilde{\zeta}_{R_a,t+1} \widetilde{R}_{a,t+1}^{*} \end{aligned} \tag{B.20}$$

Since these values are unique only for the complete markets components, they provide an alternative, decomposed view of the total cost ζ according to:

$$M_{t+1}^{*} R_{c,t+1}^{*} = M_{t+1}^{D} R_{c,t+1}^{D} \left(\zeta_{M,t+1} \zeta_{R,t+1}\right)^{-1} \tag{B.21}$$

where, as given in equation (B.20), $\zeta_{M,t+1}$ and $\zeta_{R,t+1}$ are the wedges between complete and incomplete markets for, respectively, the stochastic discount factors and the consumption asset both in domestic goods units.

B.4.2 Decomposed Wedge Valuations

In order to illustrate how each of these values are determined, consider the effect of the wedge on stochastic discount factors from $\zeta_{M,t+1}$ to a complete markets investor. Applying this wedge to equation (B.21) implies that the state price becomes: $M_{t+1}^{*} R_{c,t+1}^{*} \zeta_{M,t+1} = M_{t+1}^{D} R_{c,t+1}^{*}$. Thus, the value of this wedge can be measured in certainty equivalent units by calculating the price-to-consumption ratio Z_M that solves the Euler equation:

$$E_t \left\{ M_{t+1}^{D} R_{c,t+1}^{*} \right\} = 1. \tag{B.22}$$

for $R_{M,t+1}^{*} \equiv (C_{t+1}^{*}/C_{t}^{*})(1 + Z_{t+1}^{M})/Z_{t}^{M}$. That is, this counterfactual return is the value of a perpetual claim on domestic consumption under complete markets but valued by a domestic agent with an SDF distorted by the M-Wedge. This value can be solved using the Euler equation that considers the data-implied investor's valuation of the complete markets consumption pay-off as in:

$$E_t\left\{\beta^\theta(C^D_{t+1}/C^D_t)^{\left(-\frac{\theta}{\psi}\right)}(R^*_{c,t+1})^{\theta-1}R^*_{c,t+1}\right\}=1 \tag{B.23}$$

More generally, Table B1 summarizes the different wedges and Euler equations used to identify their wealth price ratios, Z. Given these price ratios, the implied welfare deviation from complete markets can be measured as the certainty equivalent consumption difference. Panel A sets up the structure by noting that when markets are complete, the wedge is zero. By contrast, Panel B shows the comparison of the wedges when the domestic investor is valuing wealth. Under the "Data counterpart" columns in the far left hand columns, the "Wedges" can be either the product of the M and of the R Wedges, implying the overall cost of ζ as given in equation (B.21). The successive rows show the effects of partial wedges applying only to M and to R, respectively. The next two columns under "General Pricing Relationships" provide the state price that is solved in the Euler equation to deliver a price, Z. For example, valuing the M-Wedge requires using the "data" stochastic discount factor, M^D, to value the wealth return under complete markets, R^*, as described in equation (B.23). The final two columns give the Certainty Equivalent Cost measures as compared by the price ratios, Z. Panel C lists the same information when the foreign investor is valuing the domestic wealth return, while Panel D shows the same structure for the Total S-Wedge.

Table B1: **Wedge Cost Valuation Summary**

A. Complete Markets State Price Benchmark					
Wedge		General Pricing Relationships		CE Cost relative to:	
		Data State Price	Price Ratio	Complete Mkt	Data
0		$M_t^* R_{c,t}^*$	Z^*	0	$\Delta_{D,*}$
B. Variables for Cost Decomposition in Local Prices					
Data counterpart		$M_t^* R_{c,t}^* = M_t^D R_{c,t}^D \left(\zeta_{M,t}\zeta_{R,t}\right)^{-1}.$			
Wedge	Notation	General Pricing Relationships		CE Cost relative to:	
		Data State Price	Price Ratio	Risk-Share	Data
$Total$	$\zeta_{M,t}\zeta_{R,t}$	$M_t^D R_{c,t}^D$	Z^D	$\Delta_{D,*}$	0
M	$\zeta_{M,t}$	$M_t^D R_{c,t}^*$	Z_M	$\Delta_{M,*}$	$\Delta_{D,M}$
R	$\zeta_{R,t}$	$M_t^* R_{c,t}^D$	Z_{R_c}	$\Delta_{R,*}$	$\Delta_{D,R}$
C. Variables for Cost Decomposition relative to Foreign Prices					
Data counterpart		$M_t^* R_{c,t}^* = \widetilde{M}_t^D R_{c,t}^D \left(\frac{S_t^D}{S_{t-1}^D}\right)^{-1} \left(\zeta_{\widetilde{M},t}\zeta_{R,t}\right)^{-1} (\zeta_{s,t})$			
Wedge	Notation	General Pricing Relationships		CE Cost relative to:	
		Data State Price	Price Ratio	Risk-Share	Data
η	η_t	$\widetilde{M}_t^D R_{c,t}^D (\frac{S_t^D}{S_{t-1}^D})^{-1}$	Z^η	$\Delta_{\eta,*}$	$\Delta_{D,\eta}$
S	$\zeta_{s,t}$	$\widetilde{M}_t^* R_{c,t}^* (\frac{S_t^D}{S_{t-1}^D})^{-1}$	Z_S	$\Delta_{S,*}$	$\Delta_{D,S}$
D. Variables for Cost Decomposition relative to Foreign Returns (iid only)					
Data counterpart		$M_t^* R_{c,t}^* = \widetilde{M}_t^D \widetilde{R}_{c,t}^D \left(\frac{S_t^D}{S_{t-1}^D}\right)^{-(1-\frac{1}{\gamma})} (\zeta_{s,t})^{(1-\frac{1}{\gamma})} \left(\zeta_{\widetilde{M},t}\zeta_{\widetilde{R},t}\right)^{-1}$			
Wedge	Notation	General Pricing Relationships		CE Cost relative to:	
		Data State Price	Price Ratio	Risk-Share	Data
η^T	η_t^T	$\widetilde{M}_t^D \widetilde{R}_{c,t}^D (\frac{S_t^D}{S_{t-1}^D})^{-(1-\frac{1}{\gamma})}$	Z^{η^T}	$\Delta_{\eta^T,*}$	Δ_{D,η^T}
S^T	$\zeta_{s,t}^{(1-\frac{1}{\gamma})}$	$\widetilde{M}_t^* \widetilde{R}_{c,t}^* (\frac{S_t^D}{S_{t-1}^D})^{-(1-\frac{1}{\gamma})}$	Z_{S^T}	$\Delta_{S^T,*}$	Δ_{D,S^T}
Notes:Recursive Value function determined by $V(W) = (1+Z)^{\Psi}$.					

C Complete Markets Allocations and Data Mapping

In this appendix, we highlight the solutions for the complete markets allocations based upon some standard views of exchange rate determination. These views provide examples of the applications of the framework in the text and as such are only meant to illustrate the general approach. Other applications are described briefly in Section 4 and in Appendix D.2.

C.1 General Approach

Before providing the specific examples, it is useful to describe the general approach applied to each exchange rate version. This approach follows two steps: (a) Complete Markets allocations given theoretical resources; and (b) Mapping of resources implied by the data. We therefore describe these two general steps before considering the specific examples.

C.1.1 Complete markets allocations given resources:

The general problem can be viewed as the problem of a utilitarian social planner allocating resources between consumers in two countries. Defining $Q_{t+\tau}$ and $\widetilde{Q}_{t+\tau}$ as the endogenous state price discount rates for the domestic and foreign country, respectively, between t and $t+\tau$ and Y_t and $\widetilde{Y}_t$ as the consumption-weighted index of their respective resources per period, the problem is:

$$\underset{\{C_t,\widetilde{C}_t\}}{Max}\left(U(C_t,U_{t+1})+U(\widetilde{C}_t,\widetilde{U}_{t+1})\right) \tag{C.1}$$

for

$$U(C_t,U_{t+1})=\left\{C_t^{\frac{1-\gamma}{\theta}}+\beta E_t\left[(U_{t+1})^{1-\gamma}\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}};\ U(\widetilde{C}_t,\widetilde{U}_{t+1})=\left\{\widetilde{C}_t^{\frac{1-\gamma}{\theta}}+\beta E_t\left[\left(\widetilde{U}_{t+1}\right)^{1-\gamma}\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}$$

such that wealth constraints hold:

$$E_t\sum_{\tau=0}^{\infty}Q_{t+\tau}P_{t+\tau}C_{t+\tau}=E_t\sum_{\tau=0}^{\infty}Q_{t+\tau}P_{t+\tau}Y_{t+\tau}\equiv W_t, \tag{C.2}$$

$$E_t\sum_{\tau=0}^{\infty}\widetilde{Q}_{t+\tau}\widetilde{P}_{t+\tau}\widetilde{C}_{t+\tau}=E_t\sum_{\tau=0}^{\infty}\widetilde{Q}_{t+\tau}\widetilde{P}_{t+\tau}\widetilde{Y}_{t+\tau}\equiv\widetilde{W}_t.$$

and such that goods resources clear:

$$P_t C_t + \widetilde{P}_t \widetilde{C}_t \;=\; P_t Y_t + \widetilde{P}_t \widetilde{Y}_t \equiv \widetilde{P}_t \widetilde{Y}_t^W, \forall t \tag{C.3}$$

where $\widetilde{Y}_t^W = S_t Y_t + \widetilde{Y}_t$ for $S_t \equiv \left(P_t/\widetilde{P}_t\right)$. The wealth constraints in equation (C.2) ensure that each country's agent cannot consume more than the present value of lifetime resources. Furthermore, the within-period resource constraint in equation (C.3) is written as a general constraint on the index of resources, summarized in price indices P and $\widetilde{P}$. However, the specific constraints within this general relationship depend upon the particular view of the exchange rate - Nontradeables, Home Bias, and Sticky prices - as described below.

F.O.C. with respect to wealth: The first-order condition with respect to the wealth constraint in equation (C.2) implies that the state price discount rates are equalized according to: $((Q_{t+1}P_{t+1})/(Q_t P_t)) = ((\widetilde{Q}_{t+1}\widetilde{P}_{t+1})/(\widetilde{Q}_t \widetilde{P}_t))$. Since the state price factor is the product of stochastic discount rates and prices across countries as in $Q_t \equiv \prod_{\tau=0}^{t} M_\tau \lambda_0$, this relationship gives the familiar complete markets condition given as equation (1) in the text:

$$M_{t+1}^* \left(S_{t+1}^*/S_t^*\right) = \widetilde{M}_{t+1}^*. \tag{C.4}$$

F.O.C. with respect to resources: The resource constraint in equation (C.3) gives the relationship between prices and thereby exchange rates to determine this equilibrium relationship. In turn, this condition summarizes auxiliary resources constraints depending upon the three different exchange rate views. Therefore, we describe them more fully here although we provide more details about their measures for each case later in this section.

1. Home Bias: In this case, the consumption aggregator for each country is:

$$C_t = \left(C_{1,t}\right)^a \left(C_{2,t}\right)^{1-a}; \; \widetilde{C}_t = (\widetilde{C}_{1,t})^{1-a}(\widetilde{C}_{2,t})^a \tag{C.5}$$

where $a > 1/2$ and where good 1 is preferred by domestic residents while good 2 is preferred by the foreign households. Defining the total amount of output in each good at time t as $Y_{i,t}^W$ and the allocations of resources of each good by domestic and foreign consumers as, respectively, $Y_{i,t}$ and $\widetilde{Y}_{i,t}$ for $i = 1, 2$, then the quantities embedded in the resource constraint equation (C.3) can be rewritten:

$$\begin{aligned} C_{1,t} + \widetilde{C}_{1,t} &= Y_{1,t} + \widetilde{Y}_{1,t} = Y^W_{1,t} \\ C_{2,t} + \widetilde{C}_{2,t} &= Y_{2,t} + \widetilde{Y}_{2,t} = Y^W_{2,t} \end{aligned} \tag{C.6}$$

The intratemporal first-order conditions of this constraint imply standard relative price relationships. For example, the relative quantities of goods held by domestic consumers is:

$$\begin{aligned} (C_{1,t}/C_{2,t}) \; &= a(1-a)^{-1} \left(S_t\right)^{(1-2a)} \\ \text{for } S_t &\equiv (P_t/\widetilde{P}_t) = \left(P_{1,t}/P_{2,t}\right)^{2a-1} \end{aligned} \tag{C.7}$$

with the inverse relationship holding for foreign consumers.

2. Non-Tradeables and Tradeables: Now consider the resource constraints when there are Nontradeable goods that are not exchanged in international markets. In this case, the consumption aggregator for each country is:

$$C_t \equiv (C_{T,t})^{\alpha} \left(C_{N,t}\right)^{1-\alpha} \;\; ; \widetilde{C}_t \equiv \left(\widetilde{C}_{T,t}\right)^{\alpha} \left(\widetilde{C}_{N,t}\right)^{1-\alpha} \tag{C.8}$$

As above, defining the total amount of output in each good at time t as $Y^W_{i,t}$ and the allocations of resources of each good by domestic and foreign consumers as, respectively, $Y_{i,t}$ and $\widetilde{Y}_{i,t}$ for $i = N, T$, then the resource constraint in equation (C.3) can be rewritten:

$$\begin{aligned} C_{T,t} + \widetilde{C}_{T,t} &= Y_{T,t} + \widetilde{Y}_{T,t} = Y^W_{T,t} \\ C_{N,t} = Y_{N,t} = Y^W_{N,t} \; &; \widetilde{C}_{N,t} = \widetilde{Y}_{N,t} = \widetilde{Y}^W_{N,t} \end{aligned} \tag{C.9}$$

The first-order conditions of this constraint imply standard relative price relationships for this case as well. For example, the relative quantities of goods held by domestic consumers is:

$$C_{T,t} = C_t \left(\rho_{N,t} \frac{\alpha}{(1-\alpha)}\right)^{(1-\alpha)} \;\; ; \widetilde{C}_{T,t} = \widetilde{C}_t \left(\widetilde{\rho}_{N,t} \frac{\alpha}{(1-\alpha)}\right)^{(1-\alpha)}$$

where $\rho_{N,t} \equiv (P_{N,t}/P_{T,t})$, the relative price of non-tradeables in the domestic country and similarly for $\widetilde{\rho}_{N,t}$, the relative price of non-tradeables in the foreign country. Then, according to this view, the exchange rate is:

$$S_t = \frac{(P_{T,t})(\rho_{N,t})^{1-\alpha}}{\left(\widetilde{P}_{T,t}\right)(\widetilde{\rho}_{N,t})^{1-\alpha}} = \frac{(\rho_{N,t})^{1-\alpha}}{(\widetilde{\rho}_{N,t})^{1-\alpha}} \tag{C.10}$$

where we have used the fact that $P_{T,t} = \widetilde{P}_{T,t}$ since the law of one price holds for tradeables.

3. Sticky Prices: In the case of sticky prices, the planner takes the price process as constraints. We define the "sticky" price processes that are given by this constraint as: $\overline{P}_t$ and $\overline{\widetilde{P}}_t$, for the domestic and foreign price levels, respectively. Therefore, the planner can only reallocate the resource constraints recognizing the prices are given. In this case, the resource constraint in equation (C.3) becomes:

$$\overline{P}_t C_t + \overline{\widetilde{P}}_t \widetilde{C}_t = \overline{P}_t Y_t + \overline{\widetilde{P}}_t \widetilde{Y}_t \equiv \overline{\widetilde{P}}_t \widetilde{Y}_t^W, \forall t \tag{C.11}$$

where $\widetilde{Y}_t^W \equiv \overline{S}_t Y_t + \widetilde{Y}_t$ for:

$$S_t = \overline{S}_t = \left(\frac{\overline{P}_t}{\overline{\widetilde{P}}_t}\right) \tag{C.12}$$

Clearly, this assumption represents an extreme version of sticky prices since goods markets do not adjust at all. As such, this case represents an upper bound of incomplete financial markets costs because goods markets are restricted from adjusting.

C.1.2 Mapping resources to the data

The first step described above illustrates standard complete markets allocations given three different views of exchange rate determination as examples. These solutions require knowledge of the available resources to be distributed, summarized in the variables $Y_{i,t}$ and $\widetilde{Y}_{i,t}$. Therefore, the next step is to provide a mapping from the data in order to measure the welfare costs. For this purpose, we discipline the analysis with the goods markets constraints that are presumed in each of the exchange rate views. That is, we treat the aggregate data "as if" agents are facing the same goods market conditions underlying each exchange rate determination model. In doing so, we treat aggregate consumption observations in the data as though they are the outcome of prior intertemporal financial decisions. No conditions are imposed on where these initial consumption allocations come from and therefore we do not take a stand on the degree of market completeness observed in the data. We do, however, treat the intratemporal allocations among commodities as representing the goods market conditions according to a given exchange rate view.

Take the Non-tradeables view in the data, for example. In this case, the demand for consumption of tradeables versus nontradeables is determined as a share of aggregate consumption and the relative price of nontradeables in equation (C.10). Moreover, these prices are related across countries by the relative price according to this exchange rate view as in equation (C.10). Then, *taking the consumption levels and prices from the data as given*, the commodity-level resources of tradeables and non-tradeables can be inferred by replacing aggregate consumption and prices in these equations and solving for the implied resources. As in the text, denoting measures from the data as "D", domestic tradeable consumption would be:

$$Y_{T,t} = \left(\frac{\alpha}{(1-\alpha)}\right)^{1-\alpha} \left(\widetilde{\rho}_{N,t}^{D}\right)^{1-\alpha} C_t^D S_t^D. \tag{C.13}$$

and combining with a similar expression for the foreign country, implies that aggregate world tradeables can be inferred from the data as:

$$\widetilde{Y}_{T,t}^{W} = \left(\frac{\alpha}{(1-\alpha)}\right)^{1-\alpha} \left(\widetilde{\rho}_{N,t}^{D}\right)^{1-\alpha} \left(C_t^D S_t^D + C_t^D\right) \tag{C.14}$$

Similarly, the level of non-tradeables in the data for the domestic household can be derived by substituting the solution for tradeables in equation (C.13) into the consumption aggregator in equation (C.8) and solving for $Y_{N,t}$. These substitutions also yield solutions of non-tradeables in terms of observed consumption and prices:

$$Y_{N,t} = \left(\frac{\alpha}{(1-\alpha)}\right)^{-\alpha} \left(\widetilde{\rho}_{N,t}^{D}\right)^{-\alpha} C_t^D \left(S_t^D\right)^{-\frac{\alpha}{1-\alpha}} \ , \tag{C.15}$$

with a similar solution holding for foreign non-tradeables, $\widetilde{Y}_{N,t}$. These equations then identify the consumption of non-tradeables in the data using only observed exchange rates and aggregate consumption.

By similar reasoning for the Home Bias and Sticky Price versions, we can back-out the implied levels of each good using only aggregate consumption and exchange rates. We next describe the specific solutions using these two main steps below.

C.2 Framework with Home Bias

We now describe in more detail the two-step solution for the complete markets consumption growth rates for the domestic and foreign investor.

A. Theoretical Solution of Complete Markets We first solve for the complete market shares of domestic and foreign consumption of good i, $\omega_{i,t}$ and $\widetilde{\omega}_{i,t}$, respectively. Thus, the optimal consumption allocations may be written as:

$$C_t^* = \left(\omega_{1,t} Y_{1,t}^W\right)^a \left(\omega_{2,t} Y_{2,t}^W\right)^{1-a} \tag{C.16}$$

$$\widetilde{C}_t^* = \left(\widetilde{\omega}_{1,t} Y_{1,t}^W\right)^a \left(\widetilde{\omega}_{i,t} Y_{2,t}^W\right)^{1-a} \tag{C.17}$$

We first derive the complete markets allocations given the total supply, by solving for the optimal shares determined by the relationship between the M and $\widetilde{M}$ and exchange rates in equation (C.4) using the stochastic discount factor for Epstein-Zin preferences in equation (B.11), and the fact that: $\omega_{i,t} + \widetilde{\omega}_{i,t} = 1$. Then using these solutions for the shares and defining $g_{yi,t+1} \equiv ln(Y_{i,t+1}^W / Y_{i,t+1}^W)$, the solution to the complete markets domestic and foreign consumption growth based upon these resource growth rates reduce to:

$$g_{c,t+1}^* \equiv ln(C_{t+1}^* / C_{i,t}^*) = \frac{B_2 - B_0 B_1}{1-(B_0)^2}\ g_{y1,t+1} + \frac{B_1 - B_0 B_2}{1-(B_0)^2}\ g_{y2,t+1} \tag{C.18}$$

$$g_{\widetilde{c},t+1}^* \equiv ln(\widetilde{C}_{t+1}^* / \widetilde{C}_{i,t}^*) = \frac{B_1 - B_0 B_2}{1-(B_0)^2}\ g_{y1,t+1} + \frac{B_2 - B_0 B_2}{1-(B_0)^2}\ g_{y2,t+1} \tag{C.19}$$

where the coefficients are given by:

$$B_0 \equiv \frac{2a(1-a)(1-\gamma)}{1-2a(1-a)(1-\gamma)}\ ;\ B_1 \equiv \frac{(1-a)}{1-2a(1-a)(1-\gamma)}\ ;\ B_2 \equiv \frac{a}{1-2a(1-a)(1-\gamma)}$$

Note that these consumption shares depend both upon intratemporal goods allocations, given by a, and also intertemporal consumption allocation preferences captured by γ.

B. Mapping to the Data To measure the welfare cost in the data, the next step requires solving for the available resources by viewing the consumption and exchange rate data through the lens of the model. Below, we outline a mapping to the data that applies to observed consumption growth rate processes for the two countries as well as their exchange rates. Note that equation (C.7) shows how aggregate consumption would split between the two goods depending upon the observed exchange rate. Using this condition, we can then back out the growth rate of the available commodity supplies as:

$$\ln\left(\frac{Y^W_{1,t+1}}{Y^W_{1,t}}\right) = g_{y1,t+1} \approx \left(\frac{2a(1-a)}{1-2a}\right) g^D_{s,t+1} + a g^D_{c,t+1} + (1-a) g^D_{\widetilde{c},t+1} \tag{C.20}$$

$$\ln\left(\frac{Y^W_{2,t+1}}{Y^W_{2,t}}\right) = g_{y2,t+1} \approx -\left(\frac{2a(1-a)}{1-2a}\right) g^D_{s,t+1} + (1-a) g^D_{c,t+1} + a g^D_{\widetilde{c},t+1} \tag{C.21}$$

Unlike the complete markets solution in equations (C.18) and (C.19), these measures only reflect the relative demand across goods, captured by the preference parameter a.

C. Combining Optimal Allocations with Data Measures Substituting the consumption growth rates in the data given by equations (C.20) and (C.21) into the risk-sharing consumption solution in (C.18) and (C.19) imply that the complete markets consumption growth rates in terms of the data measures as:

$$g^*_{c,t+1} = [aD_2 + (1-a)D_1]\; g^D_{c,t+1} + [(1-a)D_2 + aD_1]\; g^D_{\widetilde{c},t+1} + (D_2 - D_1)D_3\; g^D_{s,t+1} \tag{C.22}$$

$$g^*_{\widetilde{c},t+1} = [aD_1 + (1-a)D_2]\; g^D_{c,t+1} + [(1-a)D_1 + aD_2]\; g^D_{\widetilde{c},t+1} - (D_2 - D_1)D_3\; g^D_{s,t+1} \tag{C.23}$$

where the coefficients are given by:

$$D_1 \equiv \frac{B_1 - B_0 B_2}{1-(B_0)^2}\ ;\ D_2 \equiv \frac{B_2 - B_0 B_1}{1-(B_0)^2}\ ;\ D_3 \equiv \frac{2a(1-a)}{1-2a}$$

These processes are used to calculate both the optimal stochastic discount rates, M^* and $\widetilde{M}^*$, as well as the exchange rate under complete markets, S^*.

C.3 Framework with Non-tradeable goods

We now put together the same two approaches for an alternative exchange rate view that exchange rates are determined as the relative price of tradeables to nontradeables.

A. Theoretical Solution of Complete Markets We first solve for the complete market shares of domestic and foreign consumption of the tradeable good, $\omega_{T,t}$ and $\widetilde{\omega}_{T,t}$, respectively, where $\omega_{T,t} + \widetilde{\omega}_{T,t} = 1$. Only tradeables can be reallocated in this case, so that the optimal consumption allocations may be written as:

$$C_t^* = \left(\omega_{T,t} Y_{T,t}^W\right)^\alpha \left(Y_{N,t}\right)^{1-\alpha} \tag{C.24}$$

$$\widetilde{C}_t^* = \left(\widetilde{\omega}_{T,t} Y_{T,t}^W\right)^a \left(\widetilde{Y}_{N,t}\right)^{1-\alpha} \tag{C.25}$$

As with the home bias case, we first derive the complete markets allocations given the total supply, by solving for the optimal shares determined by the relationship between the M and $\widetilde{M}$ and exchange rates in equation (C.4) using the stochastic discount factor in equation (B.11). Defining $g_{y,t+1}^T \equiv ln(Y_{T,t+1}^W / Y_{T,t+1}^W)$, the solution to the complete markets domestic and foreign consumption growth based upon these resource growth rates are:

$$g_{c,t+1}^* = \ln(C_{t+1}^*/C_t^*) = \frac{1}{2}\left(g_{y,t+1}^T - (1+\frac{1}{\gamma})g_{s,t+1}^*\right) + \frac{1}{\gamma}g_{s,t+1}^* \tag{C.26}$$

$$\widetilde{g}_{c,t+1}^* = \ln(\widetilde{C}_{t+1}^*/\widetilde{C}_t^*) = \frac{1}{2}\left(g_{y,t+1}^T - (1+\frac{1}{\gamma})g_{s,t+1}^{1*}\right) \tag{C.27}$$

B. Mapping to the Data We next connect these theoretical resource measures for g_y^T and the equilibrium Complete Markets exchange rate g_s^* to the data. The world tradeables supplies is inferred using equation (C.14) while nontradeables are measured with equation (C.15). For the i.i.d. case, substituting these solutions into the optimal allocations above imply:

$$g_{s,t+1}^* = (1-\alpha)\chi(g_{c,t+1}^D - \widetilde{g}_{c,t+1}^D) - \alpha\chi g_{s,t+1}^D \tag{C.28}$$

where $\chi \equiv ((1-\alpha)\frac{1}{\gamma} - \alpha)^{-1}$. In the case of persistent risk, this relationship also depends upon the processes of price ratios, Z_t and $\widetilde{Z}_t$.

Clearly, according to the Nontradeables exchange rate view, there is an S-Wedge because a reallocation of tradeables consumption impacts the relative valuation of tradeables to nontradeables goods within each country. Specifically, the complete markets exchange rate in equation (C.28) has two terms that relate to the data. The difference between domestic and foreign consumption $g^D_{c,t+1} - \widetilde{g}^D_{c,t+1}$ that affects the equilibrium according to the share of consumption allocated to non-tradeables, $(1-\alpha)$. The quantities of this expenditure is offset by the relative price of non-tradeables observed in the data, $g^D_{s,t+1}$ according to expenditures on tradeables through α. The effects are amplified by $((1-\alpha)\frac{1}{\gamma} - \alpha)^{-1}$ because the planner cannot reallocate nontradeables. Indeed, when preferences for nontradeables are sufficiently high, the planner may not be able to reallocate tradeables in a welfare-improving way if over-all consumption is positively correlated across countries. For this reason, we assume $\alpha > 1/2$ in our examples in the text.

C.4 Framework with Sticky Prices

We now consider the benchmark when there is no exchange rate adjustment as consumption reallocates. In this case, the exchange rate process is given to the planner by equation (C.12).

A. Theoretical Solution of Complete Markets In this case, prices do not adjust within a consumption index and therefore it is isomorphic to a model with one consumption good. We therefore first solve for the complete market shares of domestic and foreign consumption bundle ω_t and $\widetilde{\omega}_t$, respectively, where $\omega_t + \widetilde{\omega}_t = 1$. In this case, the consumption allocations can be written as:

$$C^*_t = \left(\omega_t \widetilde{Y}^W_t\right) \; ; \widetilde{C}^*_t = \left(\widetilde{\omega}_t \widetilde{Y}^W_t\right) \tag{C.29}$$

where now $\widetilde{Y}^W_t = \left(\overline{S}_t Y_t + \widetilde{Y}_t\right)$. As with the earlier cases, we first derive the complete markets allocations given the total supply, by solving for the optimal shares determined by the relationship between the M and $\widetilde{M}$ and exchange rates in equation (C.4) using the stochastic discount factor in equation (B.11). The solution to the complete markets domestic consumption growth based upon these resource growth rates are:

$$g^*_{c,t+1} = \frac{1}{2} g^w_{y,t+1} - \frac{1}{2}(1 - \frac{1}{\gamma}) g^*_{s,t+1} \tag{C.30}$$

where $g^W_{y,t+1} \equiv ln(\widetilde{Y}^W_{t+1}/\widetilde{Y}^W_t)$ and where in this case, $g^*_{s,t+1} = g_{\overline{s},t+1} \equiv ln(\overline{S}_{t+1}/\overline{S}_t)$.

B. Mapping to the Data Then, under the lens of this extreme sticky price assumption, connecting these variable to the data is straightforward. Since the exchange rates are observable, the given exchange rate process must correspond to the data so that:

$$g^*_{s,t+1} = g_{\overline{s},t+1} = g^D_{s,t+1} \ . \tag{C.31}$$

Furthermore, the growth rate of world resources in numeraire prices are:

$$g^*_{y,t+1} \approx g^D_{\widetilde{c},t+1} + g^D_{c,t+1} + g^D_{s,t+1} \tag{C.32}$$

and therefore, defining $g^{wD}_{c,t+1} = g^D_{c,t+1} + \widetilde{g}^D_{c,t+1}$, we have:

$$g^*_{c,t+1} = \frac{1}{2}(g^{wD}_{c,t+1} + g^D_{s,t+1}) - \frac{1}{2}(1 - \frac{1}{\gamma})g^D_{s,t+1} \tag{C.33}$$

D Generalizations

In addition to the cases described in the text, our framework can generalize in a number of ways. Some of them we discuss in Section 4 of the text. Details are provided in this appendix.

D.1 Multiple Country Example

Incorporating more than two countries is a straightforward extension of the problem described in Appendix C. Here we describe the general framework of the extension and then provide an example based upon Sticky Prices as highlighted in the text.

General Framework with Multiple Countries: Indexing each country with superscript j, the planner now seeks to allocate resources across $j = 1, ..., J$ representative households in each country according to the problem:

$$\underset{\substack{\{C^j_t\} \\ \forall t}}{Max} \sum_{j=1}^{J} U(C^j_t, U^j_{t+1}) \tag{D.1}$$

for

$$U(C_t^j, U_{t+1}^j) = \left\{ C_t^{j\,\frac{1-\gamma}{\theta}} + \beta E_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \; j = 1, ..., J. \tag{D.2}$$

such that

$$E_t \sum_{\tau=0}^{\infty} Q_{t+\tau} P_{t+\tau}^j C_{t+\tau}^j = E_t \sum_{\tau=0}^{\infty} Q_{t+\tau} P_{t+\tau}^j Y_{t+\tau}^j \equiv W_t^j, \forall j \tag{D.3}$$

$$\sum_{j=1}^{J} P_t^j C_t^j = \sum_{j=1}^{J} P_t^j Y_t^j \equiv \widetilde{P}_t \widetilde{Y}_t^W, \forall t \tag{D.4}$$

where $\widetilde{Y}_t^W \equiv \sum_{j=1}^{J} S_t^j Y_t^j$ and where $S_t^J = 1$ for numeraire country J. As in the two country example, the solution to this problem leads to the complete markets exchange rate first-order condition as in equation (C.4). Furthermore, specific form of the goods market constraint in equation (D.4) depends upon the exchange rate view, as described above for Home Bias, Nontradeables, and Sticky Prices.

The multiple country extension requires two main modifications. First, the aggregate "world" variables are now aggregated over multiple countries and, as such, the optimized consumption growth rates incorporate the processes of all the countries. This extension can be handled in a straightforward manner by rewriting world resources available in numeraire prices as: $\widetilde{Y}_t^W$ defined above. Second, in a global economy, countries will have asymmetrical resource processes that will consequently make securities with payouts in their countries either more or less valuable relative to others. In this case, the value of wealth under complete markets will not be equal across countries.

How does this asymmetry impact the cost measures? Note that from Appendix C, the growth rates of Q, the state price discount factor, are equalized once converted into common goods prices. However, as noted there, $Q_t \equiv \prod_{\tau=0}^{t} M_\tau \lambda_0$ and therefore also depends upon an initial valuation of the wealth constraint measured in λ_0. Under our two-country symmetric model, these initial planner weights are equalized. However, more generally, the planner will allocate initial consumption to a country with more valuable wealth in order to jointly optimize global welfare. Therefore, the Certainty Equivalent costs measured in the text will have to be modified according to:

$$1 - \Delta_{D,*} = \left\{ \frac{1 + Z^D}{1 + Z^*} \right\}^{\Psi} \left(\frac{C_0^D}{C_0^*} \right) \tag{D.5}$$

where $C_0^* = \omega_0(\widetilde{Y}_0^W)$ and where $\omega_0 = \lambda_0 / \sum_{j=1}^{J} \lambda_0^j$. That is, the planner redistributes initial consumption to compensate countries with more valuable wealth.

Sticky Price Example: The text provides an example of this extension for four countries. For parsimony, we only report the Sticky-Price example as it provides an upper bound to the value of the complete markets wedges.

Table D2 reports basic relationships in the data moments for the U.S., the U.K., Canada, and Australia for the real growth rate of consumption provided in the PWT. We choose these four countries because their growth rates are similar, allowing the analysis to instead focus upon differences in second moments. Indeed, as the first line of Panel A shows, the annualized mean growth rates are all near 1.9% with Australia being on the high end during the sample. In our analysis, we set these means, captured by μ to be equal to their average. The next two lines report the standard deviations of the local prices as well as the world prices that include exchange rate variability. For these prices, Australia and the UK exhibit more volatility. Although not used in our analysis, we also report the standard deviation of prices measured at output prices for the economy in the last row of Panel A. The availability of these data provide opportunities for other extensions of our framework using production-based models, as described in more details below. Panel B of D2 gives the consumption correlation matrix. The US and Canada are clearly the most correlated, providing the rationale for our focus in the text.

Table 6 in the text then provides the consumption equivalent results for equation (D.5). In the rows, we report the results assuming first that $C_0^D = C_0^*$ with the compensation due to ω reported as "weights" underneath the row.

D.2 Other Extensions

A. Alternative Asset Pricing Models To illustrate how our framework can use consumption and exchange rates that match asset price data, we chose a long run risk process

Table D2: **Multi-Country Example Data Summary**

A. Consumption Growth				
	US	Can	UK	Aus
Mean	1.91%	1.89%	1.90%	1.96%
St Dev at Local P	1.56%	1.52%	1.76%	1.82%
St Dev at World P	1.96%	2.16%	2.82%	3.14%
St Dev at Output P	1.84%	1.97%	2.74%	2.69%
B. Consumption Correlation in Local Prices				
	US	Can	UK	Aus
US	1.00	0.57	0.45	0.08
Can	0.57	1.00	0.18	-0.02
UK	0.45	0.18	1.00	0.29
Aus	0.08	-0.02	0.29	1.00

Notes: Panel A reports the annualized means and standard deviations of consumption growth for the United States (US), Canada (Can), the United Kingdom (UK), and Australia using the Penn World Tables 9.1 through 2019. The first two lines give the means and standard deviations for consumption expenditures measured in local prices (Local P). The third and fourth rows report the same standard deviations measured at world prices (World P) and based upon country output prices (Output P), respectively. Panel B gives the cross-country correlation matrix for consumption at local prices. All prices are benchmarked to the PWT 9.1 base year.

as an example of persistent risk. However, as noted in the introduction, there are clearly other consumption-based approaches that match asset return behavior. Thus, in principle, any asset pricing model that connects returns to consumption processes can be handled in this framework.

For example, a number of papers have shown the importance of disaster risk including Barro (2009) and Gabaix (2012). Moreover, Wachter (2013) shows that consumption-measured disasters based upon Barro and Ursúa (2008) data can match U.S. asset moments in U.S. data while Backus, Chernov, and Martin (2011) show these effects using options data. Nakamura, Steinsson, Barro, and Ursúa (2013) show how disasters in different countries can be used to measure the U.S. market risk. Furthermore, Gourio, Siemer, and Verdelhan (2013) and Lewis and Liu (2017) show how disaster risk are important for explaining international risks and asset returns. Therefore, the approach in this paper could be used to consider an alternative consumption process, For example, as shown by Lewis and Liu (2017), a disaster risk process that matches cross-country asset return behavior is:

$$g_{c,t+1}^{j} = \mu + \sigma^{j}\eta_{t+1}^{j} + e^{\nu_t^j} N_t^j$$

where N is a Poisson jump process, η_t is i.i.d normally distributed, and ν is a negative random number that measures the size of the disaster. Moreover, much of this literature is based upon recursive preferences, and therefore naturally nests within our framework.

B. Persistence in Exchange Rates The endogenous price versions of the exchange rate that we have studied above require measures of the quantities within the consumption aggregate. These quantities are used to identify the resource constraints available to be redistributed through complete financial markets. In the analysis above, we have taken the simplifying assumption that the real exchange rate in the data is a random walk. However, a large literature has studied longer run behavior of the real exchange rate finding that prices do not diverge indefinitely across countries. That is, there is a stationary long run exchange rate. Thus, while the random walk assumption provides convenient closed form solutions in order to investigate the basic wedge relationships, a more realistic approach would allow for persistence in these relative prices.

To illustrate a straightforward extension to introduce this persistence in the current setting, suppose that instead of the i.i.d. exchange rate growth process in equation (C.10), this process instead contains a persistent processas in the consumption example above. That is,

$$\begin{aligned} g^D_{s,t+1} &= x_{s,t} + \sigma \nu_{s,t+1} \\ x_{s,t+1} &= \rho_s x_{s,t} + \varphi_s e_{s,t+1} \end{aligned}$$

where all random variables are normally distributed. Putting these processes together with the implied commodity allocation ratios for Home versus Foreign goods in equation (C.6) or Tradeables versus Nontradeables in equation (C.13) and (C.15) shows that the ratios of these goods will be persistent. This assumption then impacts the sharing rules and hence the long run real exchange rate under complete markets. Again, our framework is rich enough to consider these generalizations.

C. The Costs of Wedges using Production The analysis in this paper has focused upon using consumption data because it is the driver in many asset pricing models and also is an important block in most macroeconomic models. Moreover, it relates to a large literature on consumption risk-sharing. Nevertheless, the approach can easily be generalized to consider the implications of inefficient allocations in production. As described in Feenstra et al. (2015) and highlighted in the output price data in Table D2, the PWT data set provides country output and absorption price measures that can be used for international comparisons. Therefore, one could allow asset markets to span production risk rather (or in addition to) consumption risk.

One approach would be to suppose that production is linear in technology. For example, consider a model in which output in each firm is produced with linear technology:

$$y_t(z) = Y_t z l_t(z)$$

where $l_t(z)$ is the amount of labor employed by the firm and where Y_t is a stochastic process generating aggregate productivity. In this case, if domestic consumption depends upon claims to this output across countries, the total world consumption, C_t^w would be replaced by total world output in the data. In this way, the same analysis of the wedges due to real price differences across countries can be calculated for production-side risks. Rather than reallocations of consumption, the complete markets solution would instead reallocate output.

D. Alternative Exchange Rate Versions In this paper, we focused upon three different examples of exchange rate approaches. However, our approach is general enough to allow for other determination models. For example, Obstfeld and Rogoff (2001) suggested that transactions costs could potentially explain the disconnect between exchange rates and fundamentals. Given that this transactions cost approach nests within our consumption aggregator above, we can use it to provide solutions based on this exchange rate view. Specifically, they consider the aggregator for the home consumer as:

$$C_t = \left(C_{1,t}^{\Theta} + C_{2,t}^{\Theta}\right)^{\Theta^{-1}}$$

where good 1 is the domestic good and good 2 is the foreign good, as in the Home Bias case above. However, in this case, goods markets are not frictionless as consumers face iceberg shipping costs. Indeed, if these costs are proportional, the relative consumption of consumption across the two good differ according to:

$$\frac{C_{1,t}}{C_{2,t}} = \frac{\widetilde{C}_{1,t}}{\widetilde{C}_{2,t}}(1-\vartheta)^{\xi}$$

where ϱ is the proportional transaction cost and ξ is a parameter that depends upon preference parameter Θ. Since the consumption aggregator is homothetic and the transactions costs apply to goods markets, this type of goods market restriction can be readily incorporated into the framework above.

An alternative exchange rate version would be to embed the model on financial frictions directly. For example, Itskhoki and Mukhin (2021) have recently shown that a friction process outside of fundamentals are needed to explain exchange rates, a process they term "financial shocks." Given that these shocks are measured as deviations from uncovered interest parity deviations, or in our language in this paper, the foreign exchange returns, then these frictions could also potentially be treated as a measure of market incompleteness.

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